Laboratory #2: parametric estimation of a static model for a position transducer using the Set Membership approach

Introduction to the first part (29/03/2021 videotape on Teaching Portal: 0:00 - 11:00)

First part (with your PC and MATLAB R2014a, 50 minutes):

- System description
- Problem setup for a linear approximation of the sensor characteristic
- Parametric estimation of a linear model (w.r.t. data) using least squares
- Plot of the estimated approximation versus the experimental data
- Computation of the estimate uncertainty intervals EUI in l-infinity norm
- Plot of the EUI versus the estimated approximation

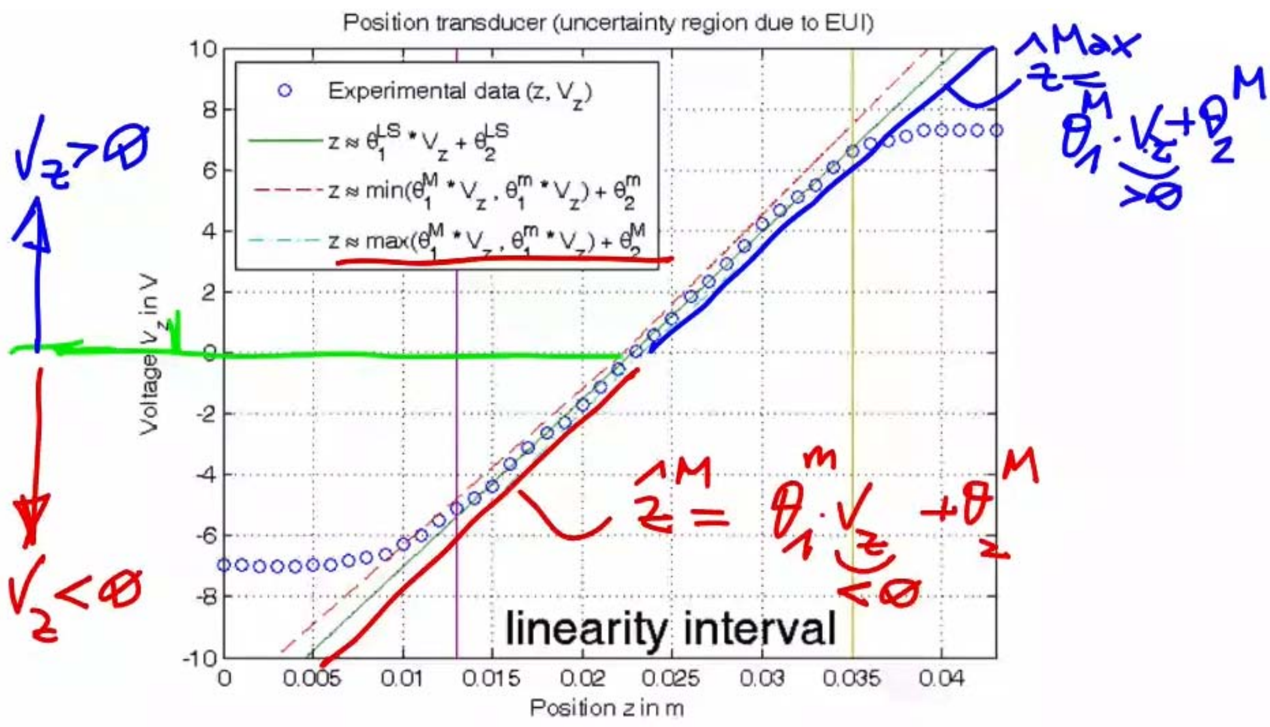
Comments on the first part (videotape: 23:00 - 44:00)

Introduction to the second part (videotape: 44:00 - 56:00)

Second part (with your PC and MATLAB R2014a, 50 minutes):

- Computation of the parameter uncertainty intervals PUI in l-infinity norm
- Computation of the parameter central estimate in l-infinity norm
- Plot of the PUI and the central estimate versus the experimental data
- (Optional) Computation of an approximation of the feasible parameter set FPS
- (Optional) Plot of PUI and FPS approximation versus the estimated parameters

Comments on the second part (videotape: 56:00 - 01:22:00)



problems. To this purpose, the MATLAB command `linprog` in its actual (R2014A) form

```
linprog(c,M,b,[],[],[],[],[],optimset(optimset('linprog'),'Algorithm','simplex'))
```

allow to solve the linear programming optimization problem

$$\min_x c^T \cdot x \quad \text{with the constraint} \quad M \cdot x \leq b$$

Note that `linprog` returns as first output argument the vector $x \in \mathbb{R}^{\dim(\theta)}$ that minimizes the objective function $c^T \cdot x$ where $c \in \mathbb{R}^{\dim(\theta)}$, i.e., not the minimum of $c^T \cdot x$. It is then necessary to suitably rewrite the inequalities that define FPS^∞:

$$\left| y_i - [\Phi \cdot \tilde{\theta}]_i \right| \leq \varepsilon \Leftrightarrow -\varepsilon \leq y_i - [\Phi \cdot \tilde{\theta}]_i \leq \varepsilon \Leftrightarrow \begin{cases} [\Phi \cdot \tilde{\theta}]_i \leq y_i + \varepsilon \\ -[\Phi \cdot \tilde{\theta}]_i \leq -y_i + \varepsilon \end{cases}, \ i = 1, \dots, N$$

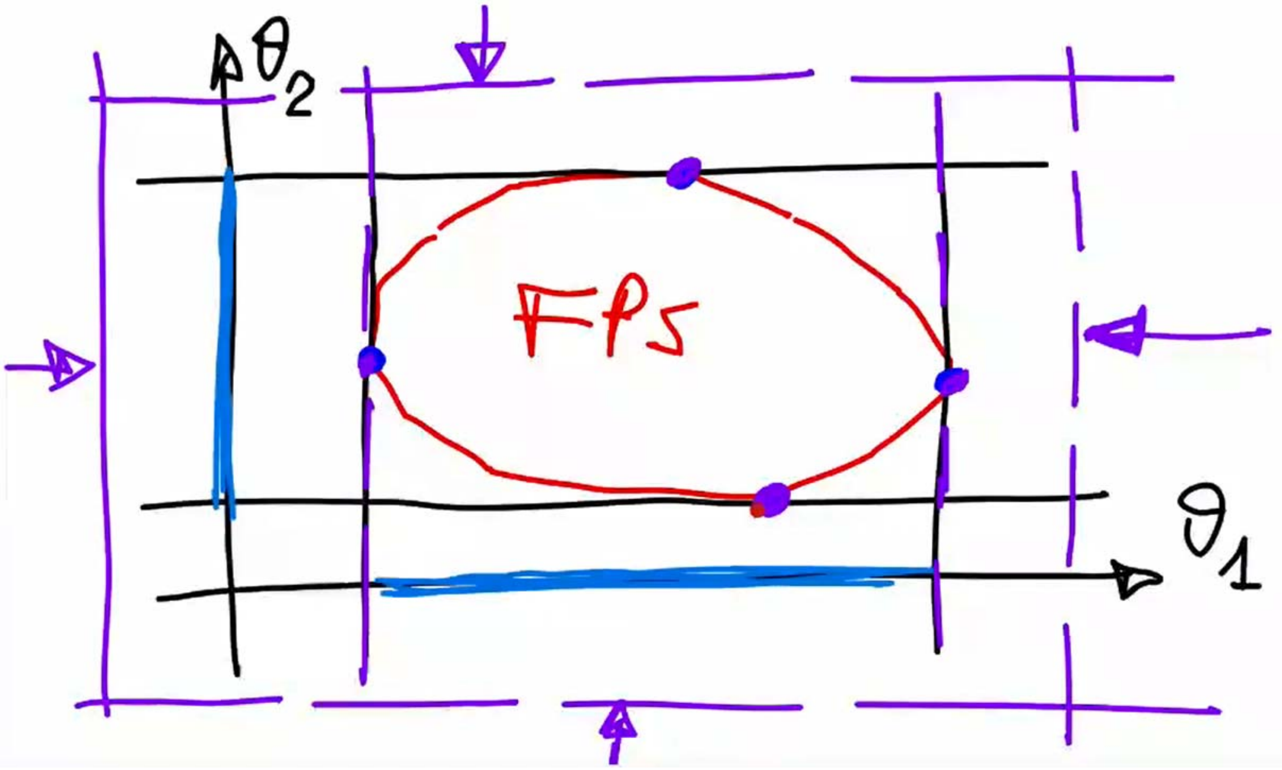
and then it follows that:

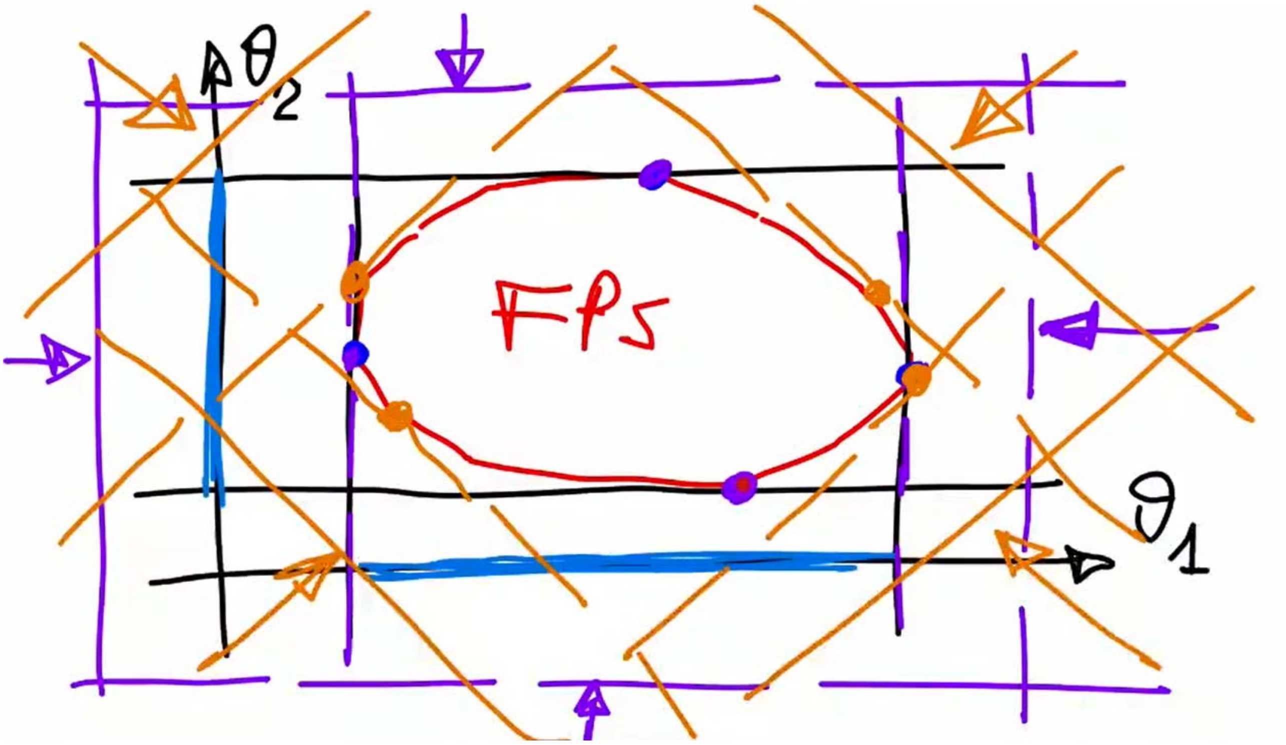
$$\theta_j^m = \min_{\theta \in FPS^\infty} \theta_j = \min_{M \cdot \theta \leq b} c^T \cdot \theta$$

$$\theta_j^M = \max_{\theta \in FPS^\infty} \theta_j = -\min_{\theta \in FPS^\infty} (-\theta_j) = -\min_{M \cdot \theta \leq b} (-c)^T \cdot \theta$$

where:

$$M = \begin{bmatrix} \Phi \\ -\Phi \end{bmatrix}, \quad b = \begin{bmatrix} y \\ -y \end{bmatrix} + \varepsilon, \quad c = j\text{-th column of the identity matrix } I_{2\times 2}$$


θ2
θ1
FPS


θ2
θ1
FPS