Exercise 2: parametric estimation of an auto-regressive model from output data

Consider the following auto-regressive (AR) dynamic model:

$$y(t) = -a_1 y(t-1) - a_2 y(t-2) + b + e(t), \quad \forall t$$

where $e(t)$ is a white noise with zero mean value and variance $\sigma_e^2 = 40$. Suppose that a set of $L = 500$ data has been generated by this model.

Problem: Estimate the parameters a_1, a_2 and b by means of Least Squares. Evaluate the quality of the obtained estimate by comparing on a plot the measured and the "predicted" outputs.

Main steps:

(1) Load the data from the file `AR_data.mat` (use `load` MATLAB command).

(2) Plot on a figure the measured data (use `plot` MATLAB command).

(3) Since the data $y(t)$ are available for $t = 1, 2, \ldots, L$, the following relationships hold:

$$\begin{cases} y(3) &= -a_1 y(2) - a_2 y(1) + b + e(3) \\ y(4) &= -a_1 y(3) - a_2 y(2) + b + e(4) \\ \vdots & \\ y(L) &= -a_1 y(L-1) - a_2 y(L-2) + b + e(L) \end{cases}$$

$\Rightarrow$ the estimation problem can be recast in the standard form:

$$y = \Phi \cdot \theta + e$$

where

$$y = \begin{bmatrix} y(3) \\ y(4) \\ \vdots \\ y(L) \end{bmatrix}, \quad \Phi = \begin{bmatrix} -y(2) & -y(1) & 1 \\ -y(3) & -y(2) & 1 \\ \vdots & \vdots & \vdots \\ -y(L-1) & -y(L-2) & 1 \end{bmatrix}, \quad \theta = \begin{bmatrix} a_1 \\ a_2 \\ b \end{bmatrix}, \quad e = \begin{bmatrix} e(3) \\ e(4) \\ \vdots \\ e(L) \end{bmatrix}$$

$\Rightarrow$ compute the Least Squares estimate $\hat{\theta}_{LS}$ of the unknown vector θ.

(4) Plot in the figure generated at step (2) the predicted output $\hat{y} = \Phi \cdot \hat{\theta}_{LS}$.

Possible solution under MATLAB (file Lab3_E2.m)

```
%% Laboratory 3 - Estimation, filtering and system identification - Prof. M. Taragna
% *Exercise 2: Parametric estimation of an auto-regressive model*
% *from output data*
%% Introduction
% The program code may be splitted in sections using the characters "%%".
% Each section can run separately with the command "Run Section"
% (in the Editor toolbar, just to the right of the "Run" button). You can do the
% same thing by highlighting the code you want to run and by using the button
% function 9 (F9). This way, you can run only the desired section of your code,
% saving your time. This script can be considered as a reference example.

clear all, close all, clc

%% Procedure
% # Load the file |AR_data.mat| containing the output data
% # Plot the experimental data
% # Estimate the model parameters using the Least Squares algorithm
% # Plot the estimated approximation versus the experimental data

%% Problem setup

% Step 1: load of data

load AR_data.mat
% y = output samples

L = length(y); % L = number of data

% Step 2: plot of data

figure, plot(1:L,y,'-b'),
axis([0,L,0,100]), grid, title('Measured data and approximation'),
xlabel('Time{\it t}'), ylabel('Output{\it y}'),
legend('Measured data',1)

%% Estimation of the model parameters using the Least Squares algorithm

% Step 3: computation of the parameter estimate

Y=y(3:L);
Phi=[-y(2:L-1), -y(1:L-2), ones(L-2,1)];
theta_LS=Phi\Y % Form #1: using the "\" operator (more realiable)

A=pinv(Phi);   % more realiable than inv(Phi'*Phi)*Phi'
theta_LS_=A*Y  % Form #2: using the pseudoinverse matrix
a1=theta_LS(1)
a2=theta_LS(2)
b=theta_LS(3)

sigma_e=sqrt(40);
Sigma_theta=sigma_e^2*inv(Phi'*Phi) % Variance matrix of the estimates
sigma_theta=sqrt(diag(Sigma_theta)) % Standard deviations of the estimates

% Step 4: graphical comparison of the results

y_LS = Phi*theta_LS;
figure, plot(1:L,y,'-b', 3:L,y_LS,'-r'),
axis([0,L,0,100]), grid, title('Measured data and approximation'),
xlabel('Time{\it t}'), ylabel('Output{\it y}'),
legend('Measured data','Least Squares estimate',1)
```