01RKYQW - Estimation, filtering, and system identification
01PDDOV/QW - Identification and Control methodologies
Sample examination paper (exam duration: 3 hours)

SURNAME: ______________________ **NAME:** ______________________

Remark: making use of MATLAB R2014A only, write two stand-alone m-files (named `es1.m` *and* `es2.m`*) to solve the problems, respectively. Write by hand a clear report that includes the reasoning behind the computations, the main numerical results and their possible critical analysis.*

Problem #1: the input-output measurements of a SISO dynamic system $\mathcal{S}_1$ to be modeled have been collected in the MATLAB `data1.mat` file.
1) Identify ARX, ARMAX and OE models of different orders and delays, using only a part of the experimental data and looking for models that guarantee satisfactory characteristics of whiteness of the residuals associated to this first dataset.
2) Compare the identified models on a different set of data not used for identification. To assess the model quality, minimize the Root Mean Square Error $RMSE = \sqrt{\frac{1}{N-N_0}\sum_{t=N_0+1}^{N}[y(t)-\hat{y}(t)]^2}$, where $y(t)$ = measured output, $\hat{y}(t)$ = simulated (or predicted) output and N_0 is a suitable time instant after which the transient is past. Choose the best trade-off between $RMSE$ and model complexity: $n = n_a + n_b$ (for ARX models), $n = n_a + n_b + n_c$ (for ARMAX models) and $n = n_f + n_b$ (for OE models), if the model is used for prediction purposes; $n = n_a$ (for ARX and ARMAX models) and $n = n_f$ (for OE models), if the model is used for simulation purposes.
3) Using all the experimental data, estimate the parameters of an $ARX(3,3,1)$ model by means of the standard Least-Squares algorithm and compare:
- the Estimate Uncertainty Intervals EUI^2 and, if possible, the Parameter Uncertainty Intervals PUI^2, assuming that the output measurements are corrupted by an energy-bounded noise whose 2-norm is less than 4;
- the Estimate Uncertainty Intervals EUI^∞, assuming that the output measurements are corrupted by an amplitude-bounded noise whose ∞-norm is less than 0.1.

Problem #2: consider the following LTI dynamic system $\mathcal{S}_2$:

$$\begin{aligned} x(t+1) &= Ax(t) + Bu(t) + v_1(t) \\ y(t) &= Cx(t) + v_2(t) \end{aligned}$$

where

$$A = \begin{bmatrix} 2.7258 & -1.2424 & 0.7577 \\ 2 & 0 & 0 \\ 0 & 0.5 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0.026 \\ 0 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & -0.45 & -0.05 \end{bmatrix}$$

$v_1(t)$ is a white noise with zero mean value and variance $V_1 = B_{v_1}B_{v_1}^T$, $B_{v_1} = 0.05\begin{bmatrix} 0.026 & 0 & 0 \end{bmatrix}^T$, $v_2(t)$ is a white noise with zero mean value and variance $V_2 = 0.0004$ and $u(t)$ is a suitable input signal whose values have been saved in the MATLAB `data2.mat` file. The noises $v_1(t)$ and $v_2(t)$ are uncorrelated. Assume that the initial state $x(1)$ is a random vector with zero mean value and variance $P_1 = E\left[x(1)x(1)^T\right] = I_3$.
1) Design the steady-state Kalman filter $\mathcal{F}_\infty$ and the dynamic Kalman 1-step predictor in predictor-corrector form $\mathcal{K}_{pc}$.
2) Compare the state estimates provided by $\mathcal{F}_\infty$ and $\mathcal{K}_{pc}$ by means of graphical representations and evaluate the Root Mean Square Errors:

$$RMSE_k = \sqrt{\frac{1}{N'-N'_0}\sum_{t=N'_0+1}^{N'}[x_k(t)-\hat{x}_k(t)]^2}, \quad k = 1,\dots,3$$

where $\hat{x}_k(t)$ is the estimate of the state $x_k(t)$ and N'_0 is a suitable time instant after which the filter or predictor transient is past.