



# CONTROL AND OPTIMIZATION IN SMART-GRIDS

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# Course topics

- Session 1: Introduction to Power systems
  - Context and motivation
  - Power flow analysis
  - Economic dispatch
- **Session 2: Renewable sources**
  - **Stochastic models of variable sources**
  - **Dispatching random sources**
- Session 3: Energy dispatch
  - Risk-limiting dispatch
  - Matlab session



# Course topics



- Session 4: Incentive-based demand response
  - Modeling demand
  - Peak time rebates
  - Contract design for demand response
- Session 5: Flexible loads
  - Modeling flexibility
  - Load dispatch
  - Case study: Electric vehicles
- Session 6: Micro-grids
  - Lean energy concept
  - Joint generation and load dispatch



# Day-ahead Market

- Given a demand forecast  $D_k$
- And a set of generators  $G_1, G_2, G_3, \dots, G_N$
- What is the lowest cost generation program that supplies the demand?

*This is the economic dispatch problem!*



# Economic Dispatch

$$\min J = \sum_{k=1}^K \sum_{j=1}^N C(p_{jk})$$

*Sum of generation costs*

*Subject to*

$$\sum_{j=1}^N p_{jk} = D_k \quad \forall k$$

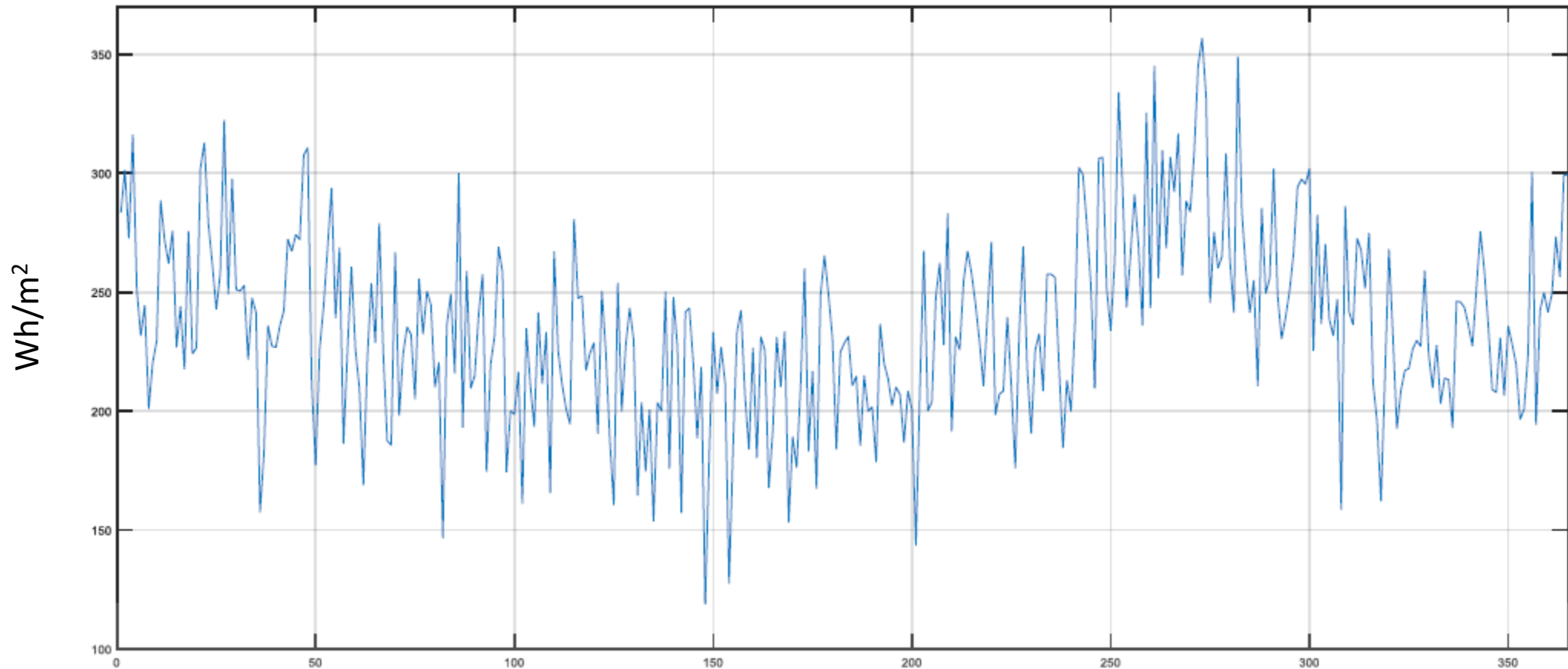
Energy Balance

$$p_j^{\min} \leq p_{jk} \leq p_j^{\max}$$

Operational constraints



# Renewable energies



Daily average of solar radiation for one year at Medina (Colombia)



# Economic Dispatch

$$\min J = \sum_{k=1}^K \sum_{j=1}^N C(p_{jk})$$

Subject to

$$\sum_{j=1}^N p_{jk} = D_k \quad \forall k$$

$$p_j^{\min} \leq p_{jk} \leq p_j^{\max}$$

- Can we use the same approach to dispatch renewable sources?
- Can we fix  $p_j$ ?

*Give possible solutions...*



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# Renewable energies



- Renewable power generation can be modeled as a stochastic process!





# Stochastic Process

Unknown signals: Can be described by their statistical properties.

$v(t, x, y, z)$  is an stochastic process:

- *Sequence of random variables*
- *Continuous ( $v \in \mathbb{R}$ ) or discrete ( $v \in S$ )*
- How is it fully described in stochastic sense?

$$F_{(X(t_1), \dots, X(t_k))}(x_1, x_2, \dots, x_k) \rightarrow \text{joint cdf}$$
$$F_{(X(t_1), \dots, X(t_k))}(x_1, x_2, \dots, x_k) = P[x(t_1) \leq x_1, \dots]$$



# Stochastic Process

Given:

$$F_{(X(t_1), \dots, X(t_k))}(x_1, x_2, \dots, x_k) = \text{joint cdf}$$

The probability density function (pdf) is:

$$f_{(X(t_1), \dots, X(t_k))}(x_1, x_2, \dots, x_k) = \frac{\partial^n F_{(X(t_1), \dots, X(t_k))}(x_1, x_2, \dots, x_k)}{\partial x_1 x_2 \dots x_k}$$

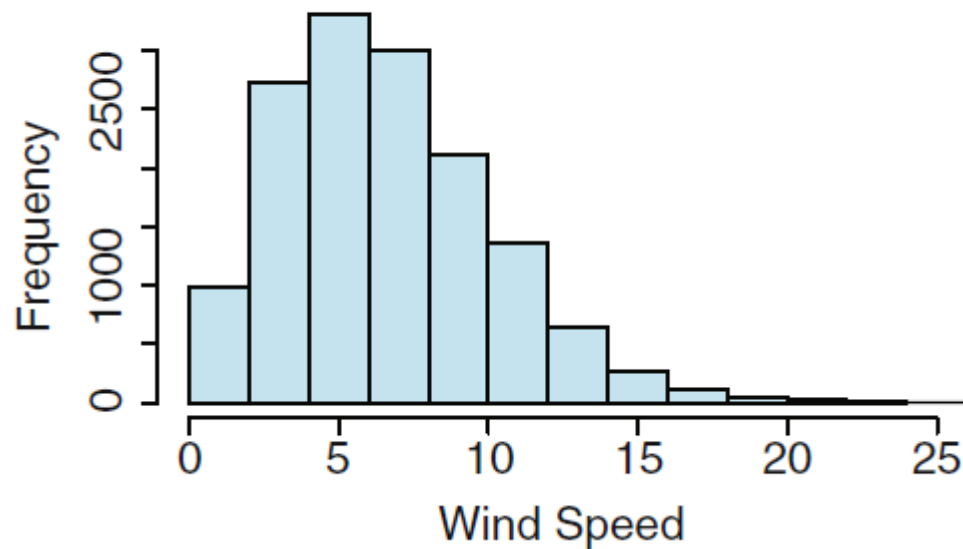
–It is a highly complex representation

–The *pdf* is infinite dimensional, for any time  $k$ , for any space point,...

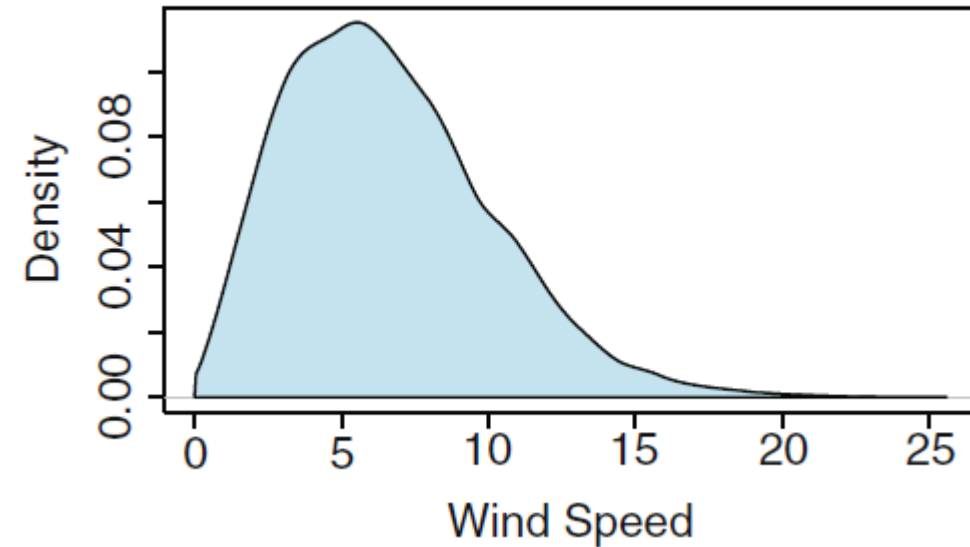


# Stochastic Process

Histogram



Non-parametric density



Descriptive *pdf* of wind speed in east-Denmark constructed from 1970-1979 data



# Stochastic Process

Given:

$$f_{(X(t_1), \dots, X(t_k))}(x_1, x_2, \dots, x_k) = \frac{\partial^n F_{(X(t_1), \dots, X(t_k))}(x_1, x_2, \dots, x_k)}{\partial x_1 \partial x_2 \dots \partial x_k}$$

- First moments:

- Expected value

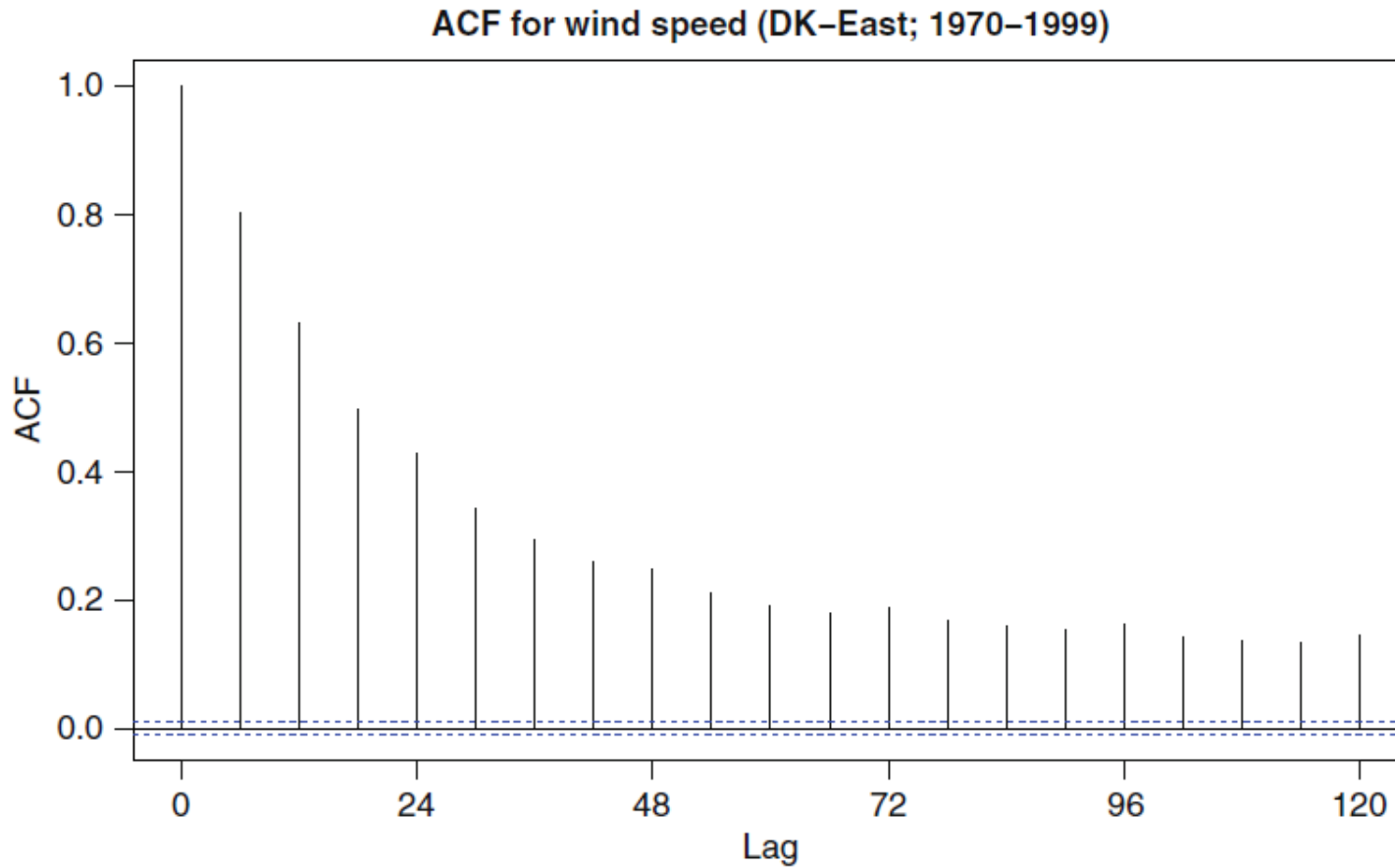
$$\mu_X(t_k) = E_X(t_k) = \int_{-\infty}^{\infty} x f_{X(t_k)}(x; t_k) dx$$

- Covariance

$$COV_{XX}(t_k, t_l) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (x_k - E[x_k])(x_l - E[x_l]) f_{(X(t_k), X(t_l))}(x_k, x_l; t_k, t_l) dx_k dx_l$$



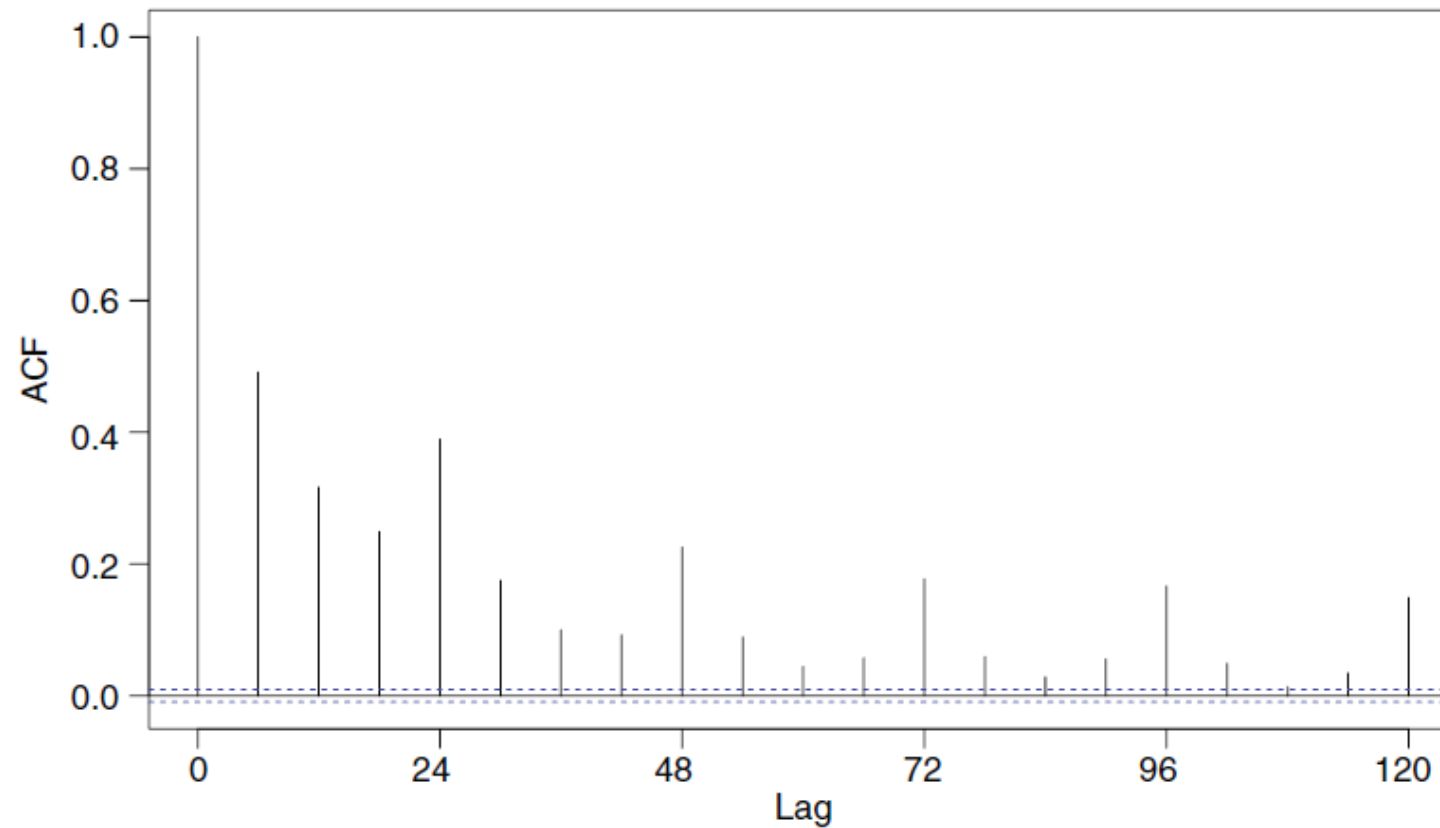
# Stochastic Process





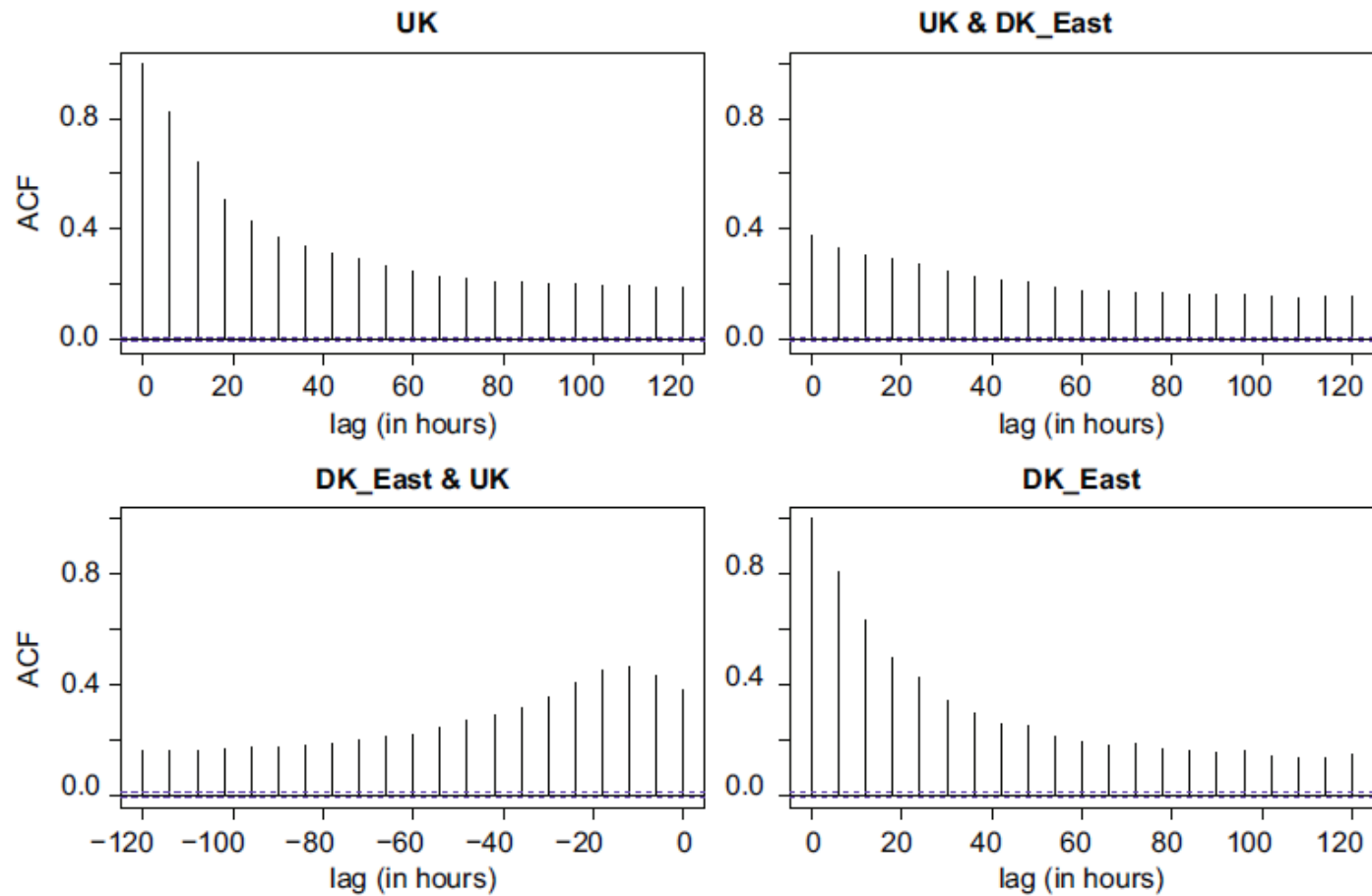
# Stochastic Process

ACF for wind speed (Spain; 1970–1999)





# Stochastic Process





# Quantiles

- Given a probability  $\alpha$ , what is the value of the random variable  $y$  such that:

$$P[Y < y] = \alpha$$

That is:

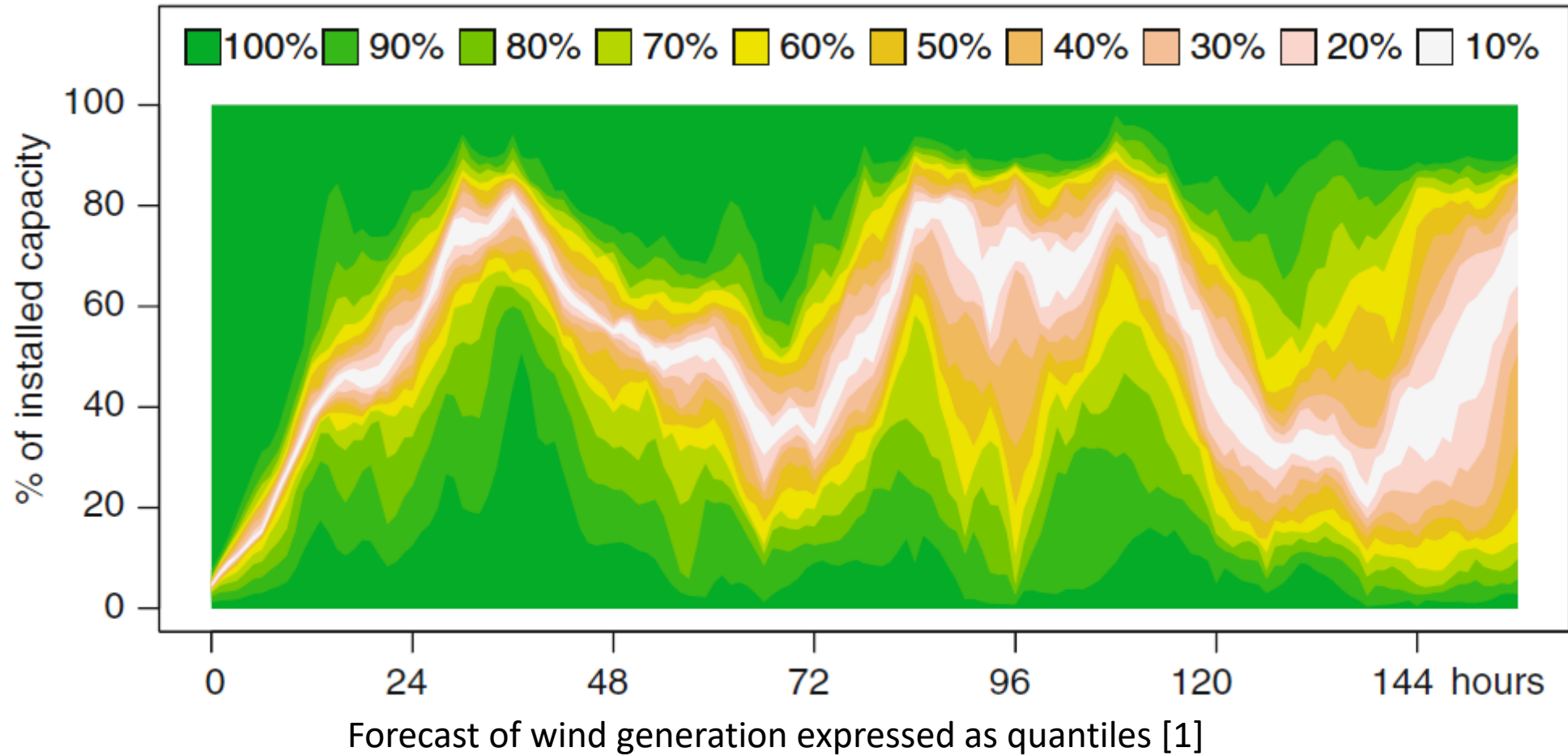
$$q(\alpha) = F^{-1}(\alpha)$$

$F$  is an (strictly) increasing function of  $y$ .





# Quantiles





# Bayesian Estimation

How to obtain an optimal generation forecast from a probabilistic description of the renewable source?



# Bayesian Estimation

- The unknown is NOT a parameter
- There is INFORMATION in the form of observed realizations of random variables
- Both the unknown and the observed information are described as random variables
- The unknown and the observed R.V. are Correlated.
- Estimation criteria:
  - Minimize Bayesian Risk
  - Minimize Mean Squared Error



# Mean Squared Error

- Given an estimate  $X'$  of  $X$  (random variable), it is defined:
- Estimation error:

$$X = X - X'$$

- Mean squared error (Expected cost)

$$J = E [ X^T X ] = E [ (X - X')^T (X - X') ]$$

## **MSE estimate: Minimize the mean squared error**

- *We want to select an estimate  $X'$  such that  $J$  is minimized, using all the available information.*



# Mean Squared Error

- SITUATION 1:

The unique available information is :

$f_X(x)$ , pdf of  $X$

What is the optimal MSE estimate of  $X$ ?

- $\mathbf{J} = E [ X^T X ] = E [ (X-X')^T (X-X') ]$



# Mean Squared Error

- SITUATION 1:

The unique available information is :

$f_X(x)$ , pdf of  $X$

What is the optimal MSE estimate of  $X$ ?

- $J = E [ X^T X ] = E [ (X-X')^T (X-X') ]$

Answer:

$$X' = E[X]$$

What is the variance of this estimate?



# Mean Squared Error

- SITUATION 2:

We are given the realization of another random variable  $Z$ , jointly distributed with the unknown  $X$ .

$f_{XZ}(x,z)$ , joint pdf of  $X$  and  $Z$

What is the optimal MSE estimate of  $X$ ?

- $J = E [ X^T X ] = E [ (X-X')^T (X-X') ]$



# Mean Squared Error

- SITUATION 2:

We are given the realization of another random variable  $Z$ , jointly distributed with the unknown  $X$ .

$f_{XZ}(x,z)$ , joint pdf of  $X$  and  $Z$

What is the optimal MSE estimate of  $X$ ?

- $J = E[X^T X] = E[(X-X')^T(X-X')]$

Answer:

$$X' = E[X/Z]$$

The optimal Bayesian estimate is the conditional mean!!





# Mean Squared Error

- The conditional pdf can be expressed as:

$$f[X | Z] = \frac{f(Z | X) f(X)}{f(Z)}$$

- And the conditional mean is obtained as:

$$E[X/Z] = \int x f_{X/Z}(x/z) dx$$



# Mean Squared Error

A property of any MMSE estimator is that for any function  $g(Z)$  of the observed R.V. it holds that:

$$E[g(Z) (X - E[X | Z])^T] = 0$$

Then:

$$E[ \|X - E[X|Z]\| ] \leq E[ \|X - g(Z)\| ]$$



# Renewable generation forecast

- Point forecast:
  - For the economic dispatch we are interested in the energy produced by a point source  $y$  at a future time  $t+k$ , given observations up to present time  $t$ .

- The optimal point estimate is the conditional mean:

$$\hat{y}(t+k|t) = E[Y(t+k) | \Omega(t)]$$

- Where  $\Omega(t)$  is the information set at time  $t$ .



# Renewable generation forecast

$\Omega(t)$  contains all data and knowledge of the process up to time  $t$ .

For example:

- Variable realization for previous intervals:  $y(t), y(t-1), y(t-N)$
- Correlated variables realizations (weather, generation at close locations,...)

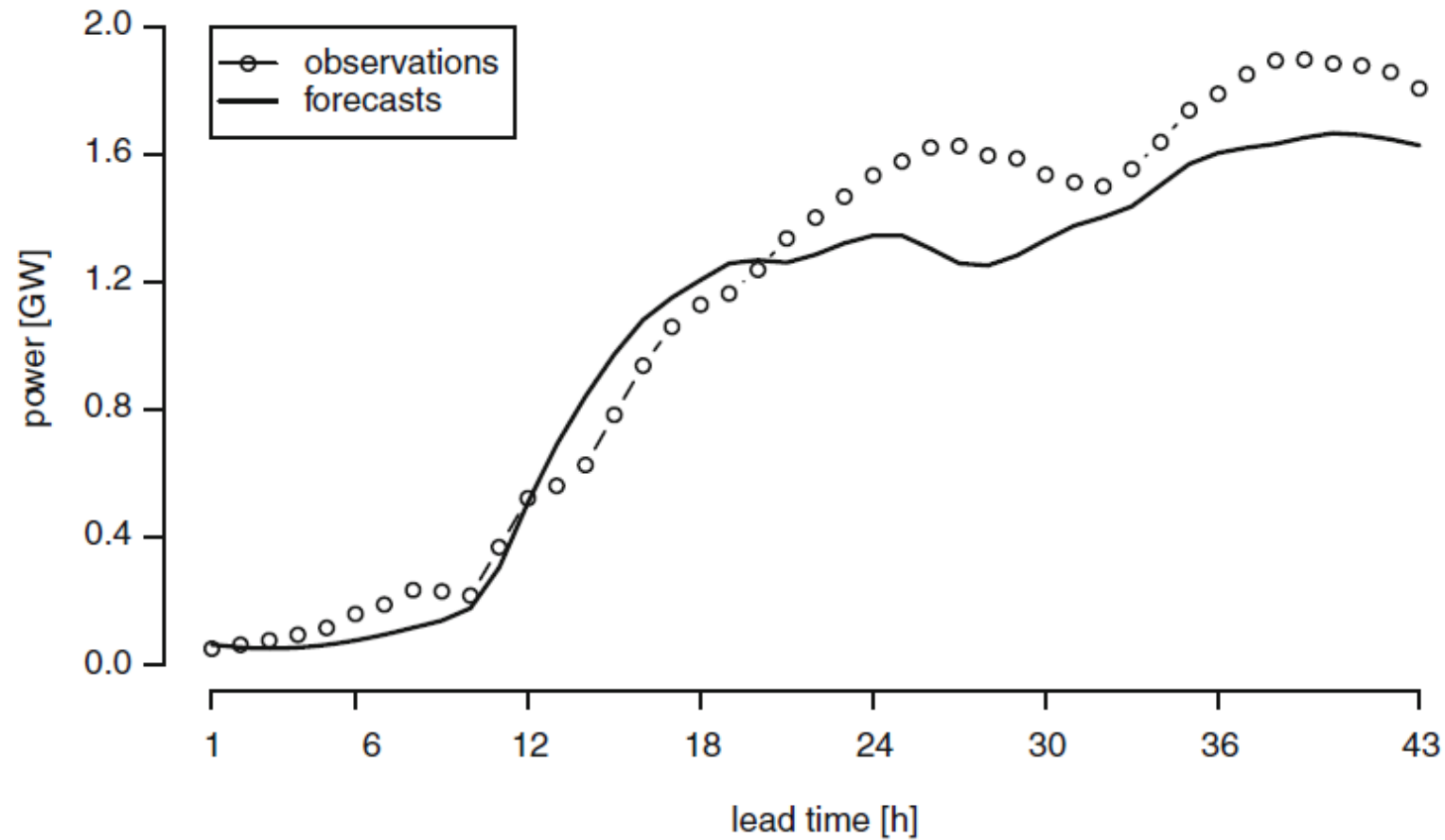
- Note that , for a known joint pdf

$$\hat{y}(t+k|t) = E[Y(t+k)|\Omega(t)]$$

Is a function of  $\Omega(t)$



# Renewable generation forecast



Point estimate and observations of wind generation in western Denmark, 4<sup>th</sup> April 2007.



# Renewable generation forecast

Point estimates DO NOT give information on uncertainty levels!

Better instruments can be derived:

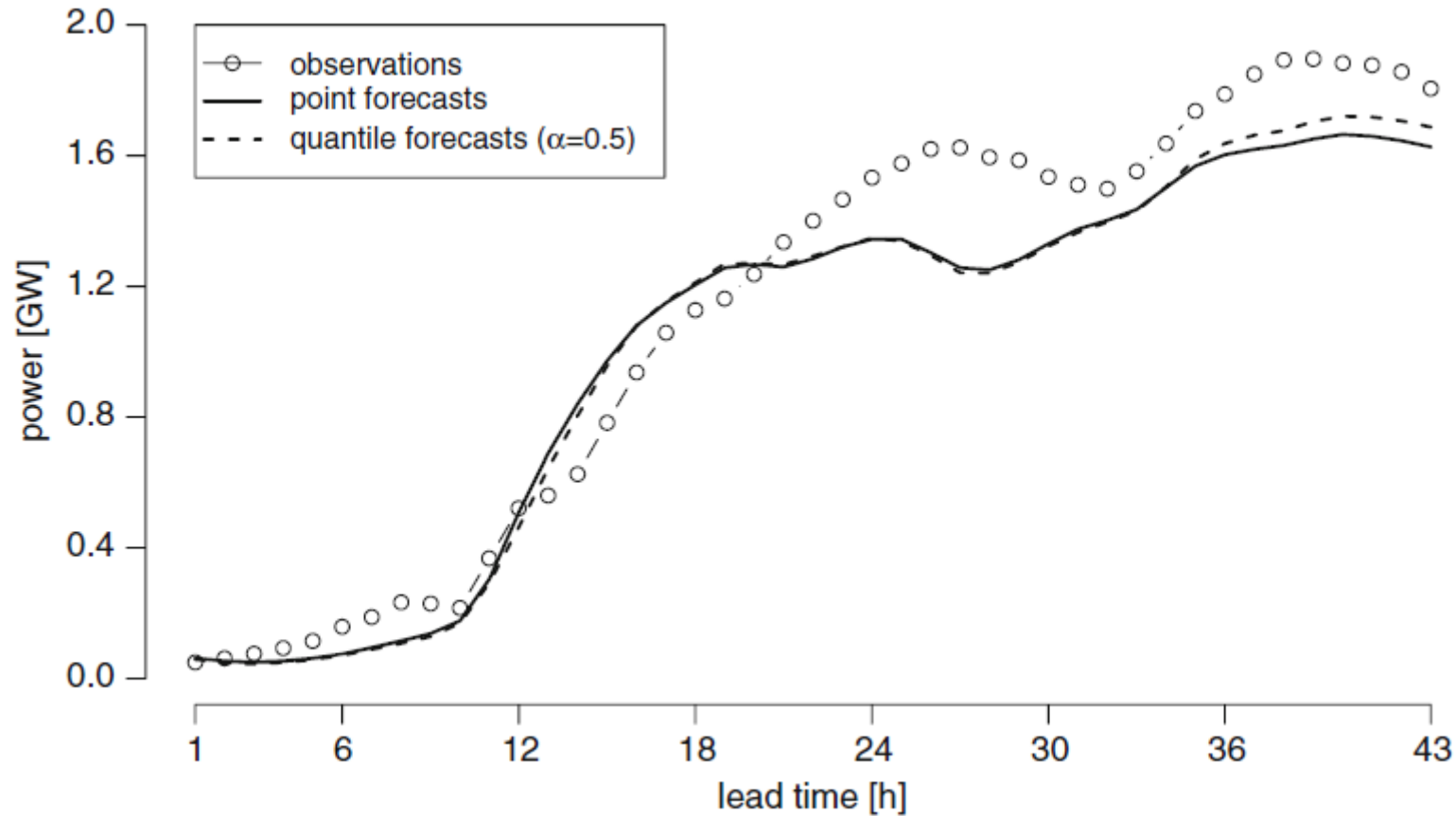
- **Quantile forecasts:**  $q^\wedge(t+k, \alpha)$

is the R.V. value such that:

$$P[ Y(t+k) \leq q^\wedge(t+k, \alpha) \mid \Omega(t) ] = \alpha$$



# Renewable generation forecast



0.5 quantile estimate and observations of wind generation in western Denmark, 4<sup>th</sup> April 2007.



# Renewable generation forecast

- **Prediction intervals:** A prediction interval is a range of possible outcomes for the variable  $Y(t+k, \beta)$ , given the present information  $\Omega(t)$ , for a level of probability  $(1 - \beta)$ :

$$P[ Y(t+k) \in I^\wedge(t+k, \beta) \mid \Omega(t) ] = 1 - \beta$$

- where

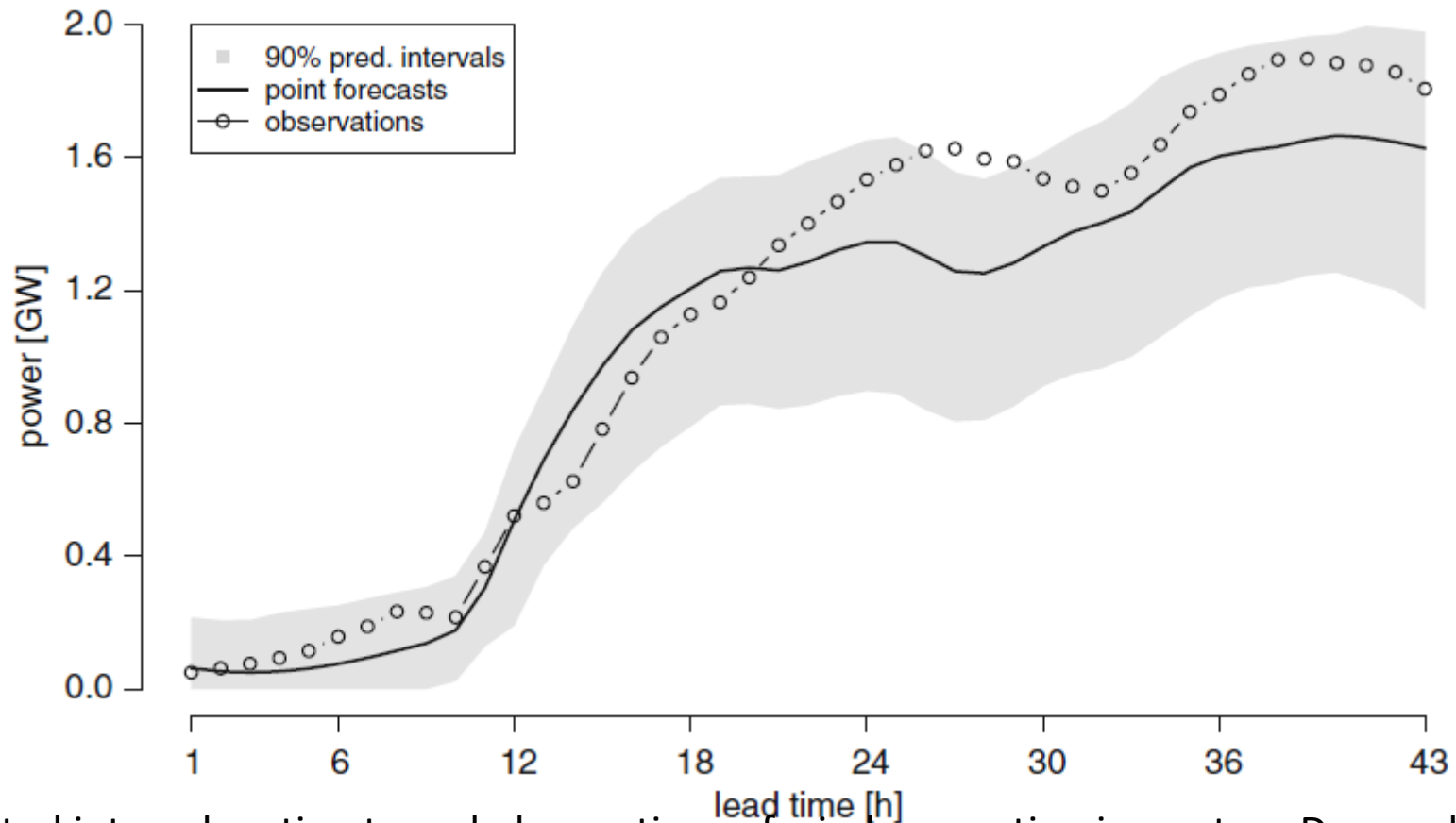
$$I^\wedge(t+k, \alpha) = [q^\wedge(t+k, \alpha_{min}); q^\wedge(t+k, \alpha_{max})]$$

- *Note that the intervals are not unique for a given provability level.*
- *Central intervals are usually employed (centered on the median).*





# Renewable generation forecast



10% central intervals estimate and observations of wind generation in western Denmark, 4<sup>th</sup> April 2007.



# Renewable generation forecast

## Scenario approach:

- A pdf can be represented by samples of the random variable.

Being  $Y$  a random process, a realization (sample) is a sequence:

$$Y^{\wedge}(t,j)=[y(t+1,j), y(t+2,j), \dots, y(t+k,j)],$$

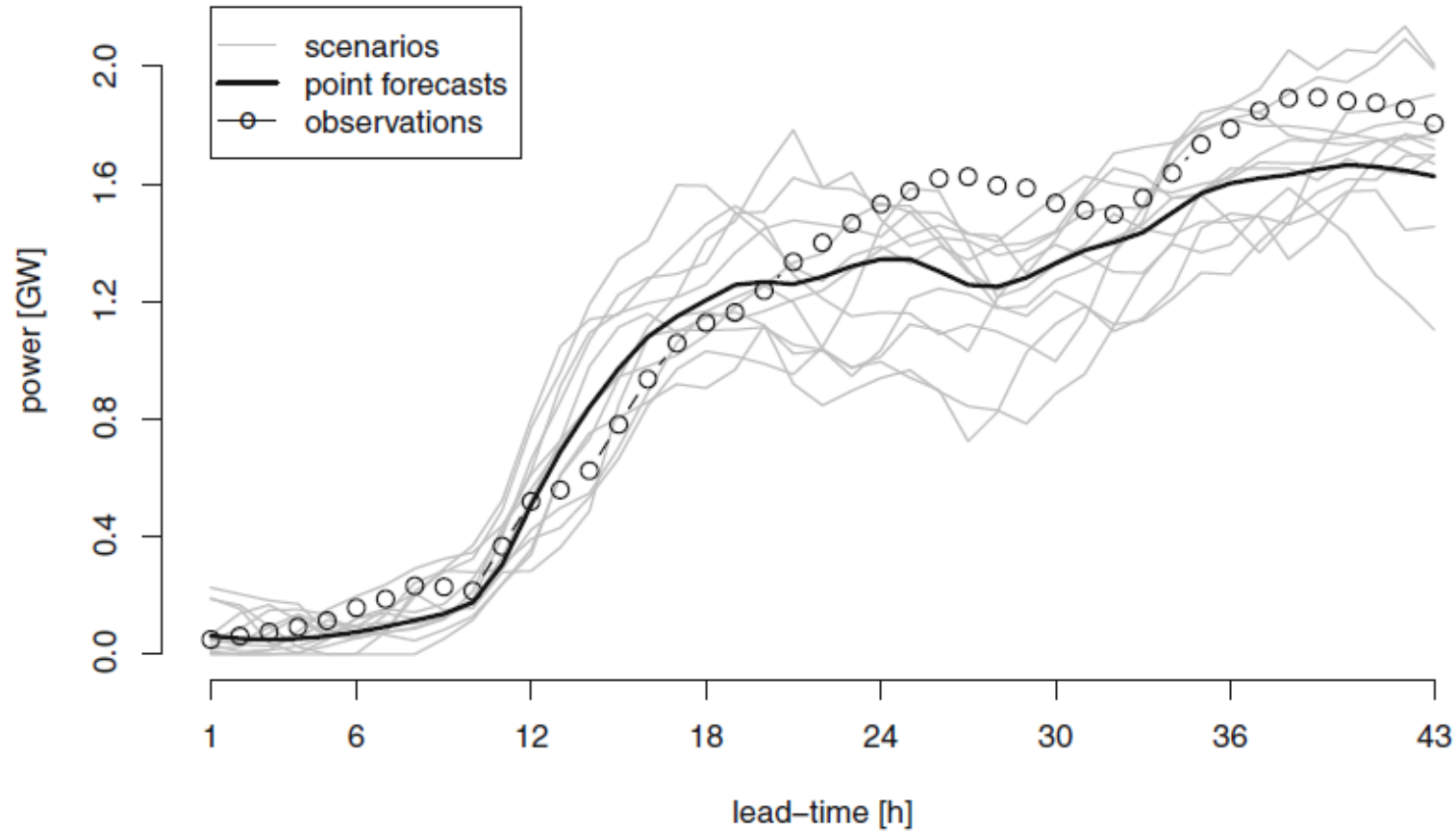
where

$y(t+i,j)$  is a sample of the pdf  $f(y(t+i) | \Omega(t))$

- As the number  $j$  of samples grows, the scenarios are a precise representation of the pdf.
- This approach is useful very in stochastic programming (more on this later)



# Renewable generation forecast



Observations and 12 scenarios of wind generation in western Denmark, 4<sup>th</sup> April 2007.



# Case study



## Solar radiation estimation

- Isolated micro-grid
- Solar panels are the only energy source
- Next day generation is required to plan energy management:
  - Battery charging scheduling
  - Interruptible services
  - Unserved load
- Limited computational resources





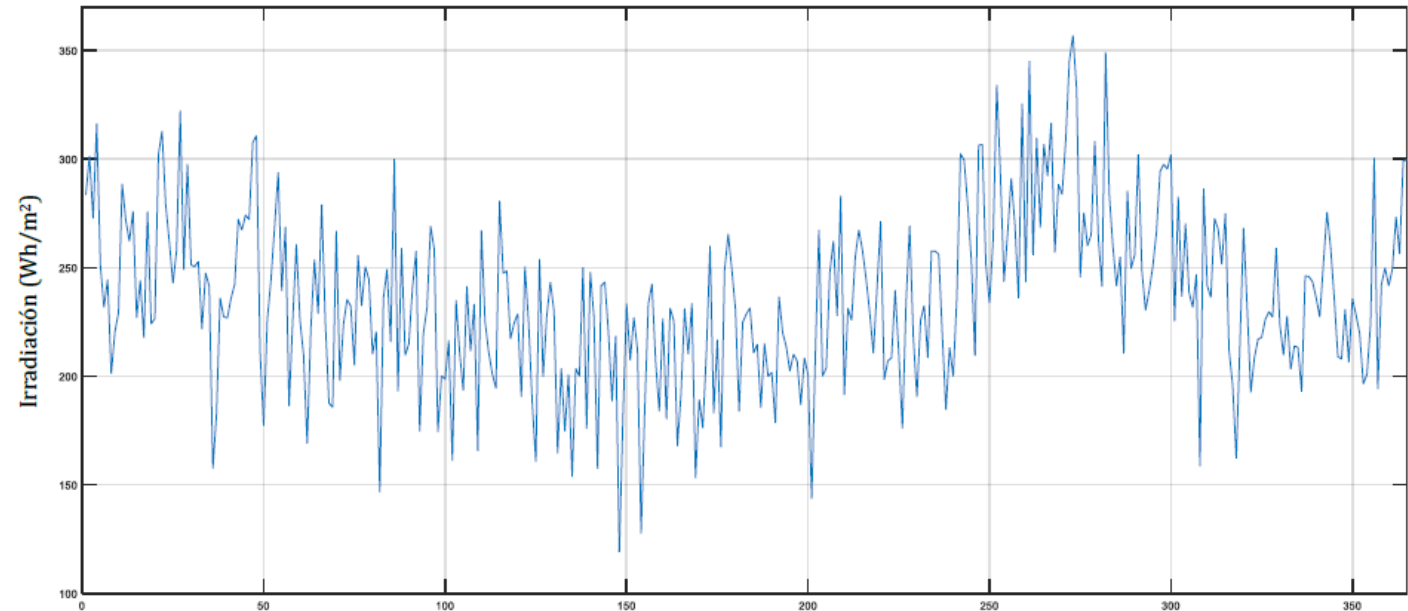
# Case Study

## Available data:

- Solar irradiation (insolation) in  $\text{Wh}/\text{m}^2$ .
- Hourly measurements between June 2008 and December 2014.

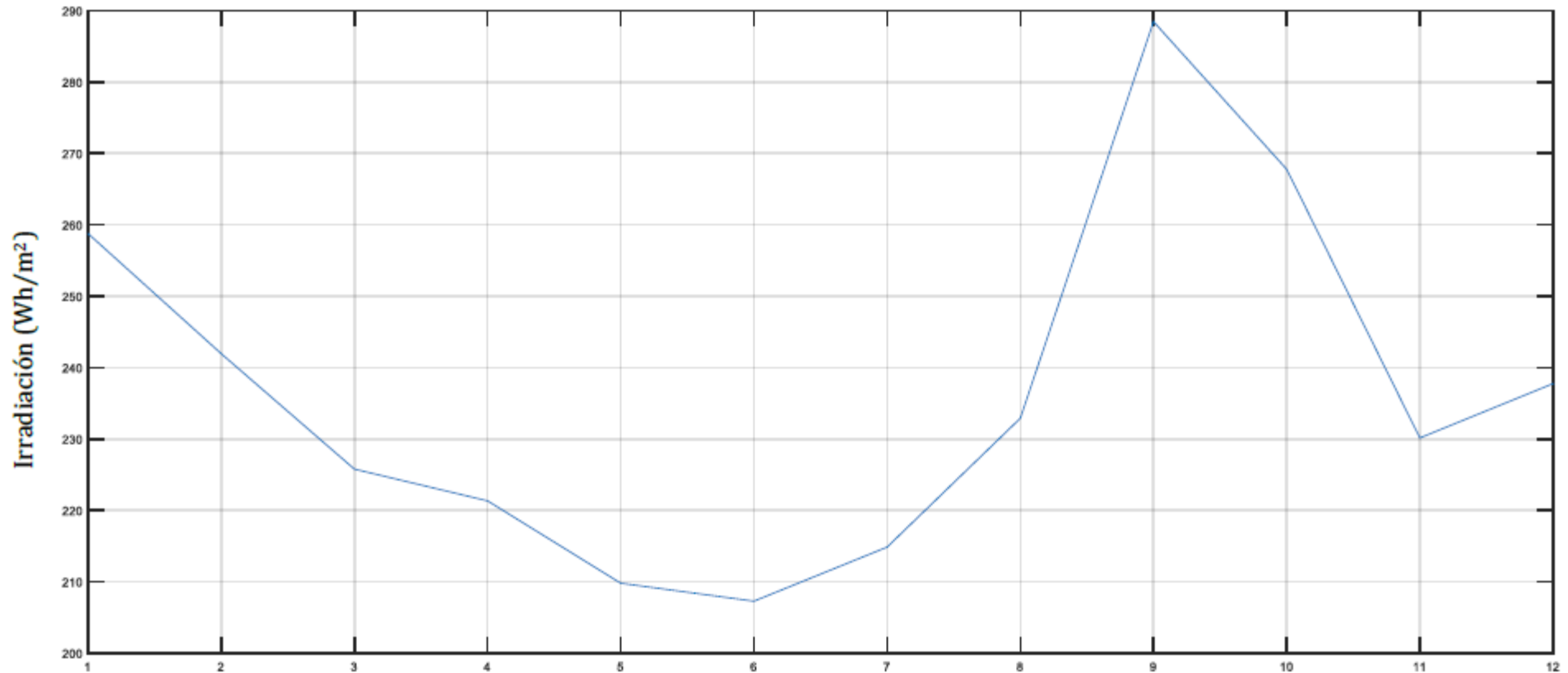
## Required estimates:

- Insolation for the next 12 hours (day)





# Case Study



Monthly average of insolation for the complete data set.



# Case study

Linear auto-regressive models:

- AR:

$$y(t) = \alpha_0 + \alpha_1 y(t-1) + \alpha_2 y(t-2) + \dots + \alpha_p y(t-p) + \varepsilon(t)$$

- ARMA:

$$y(t) = \alpha_0 + \alpha_1 y(t-1) + \alpha_2 y(t-2) + \dots + \alpha_p y(t-p) + \varepsilon(t) + \vartheta_1 \varepsilon(t-1) + \vartheta_2 \varepsilon(t-2) + \dots + \vartheta_q \varepsilon(t-q)$$

- $\varepsilon(t)$  is assumed as white noise.



# Case study



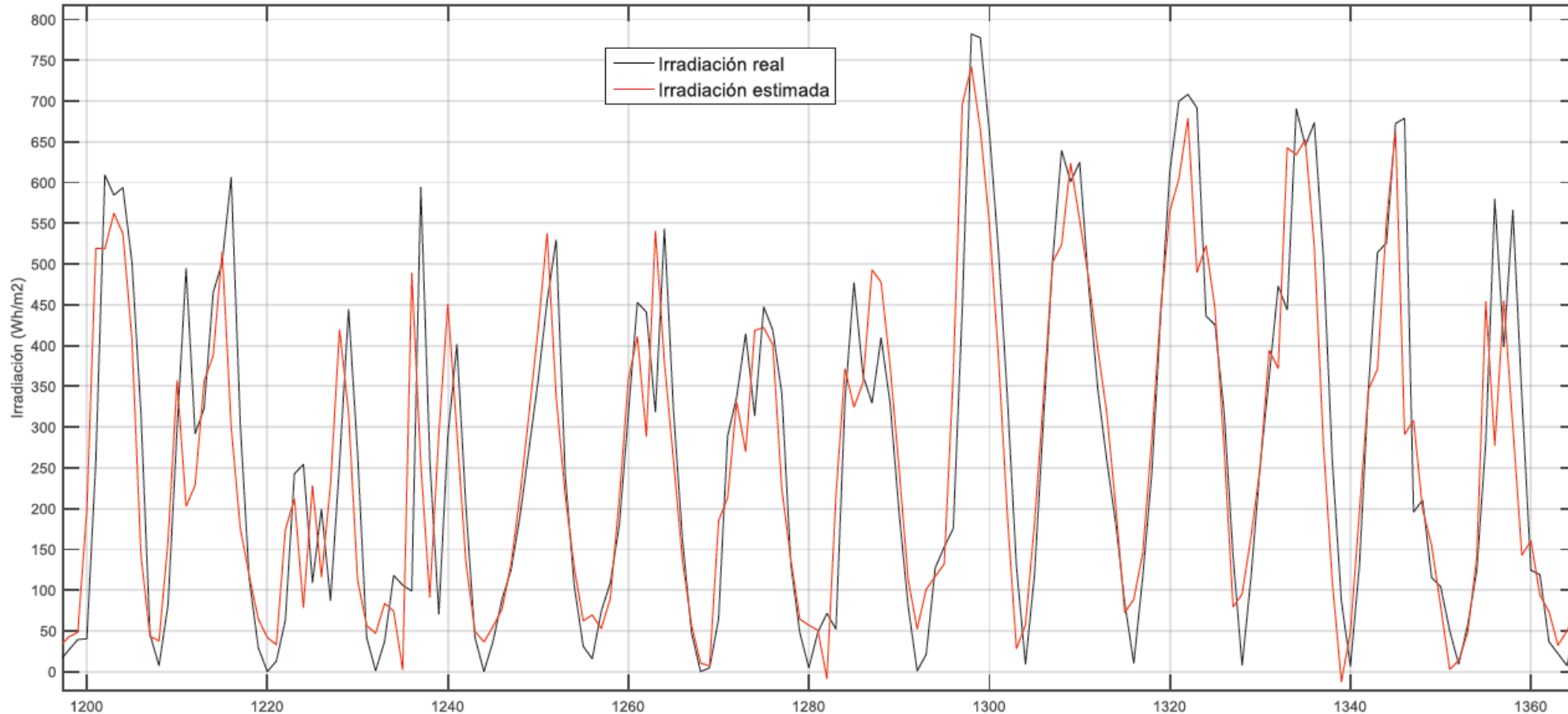
## Data preprocessing:

- Eliminate outliers
- Fill missing data
- Select estimation set (70% of available data)
- Adjust data to solve least squares problem
- Estimate coefficients  $\theta_i$  and  $\alpha_i$ .





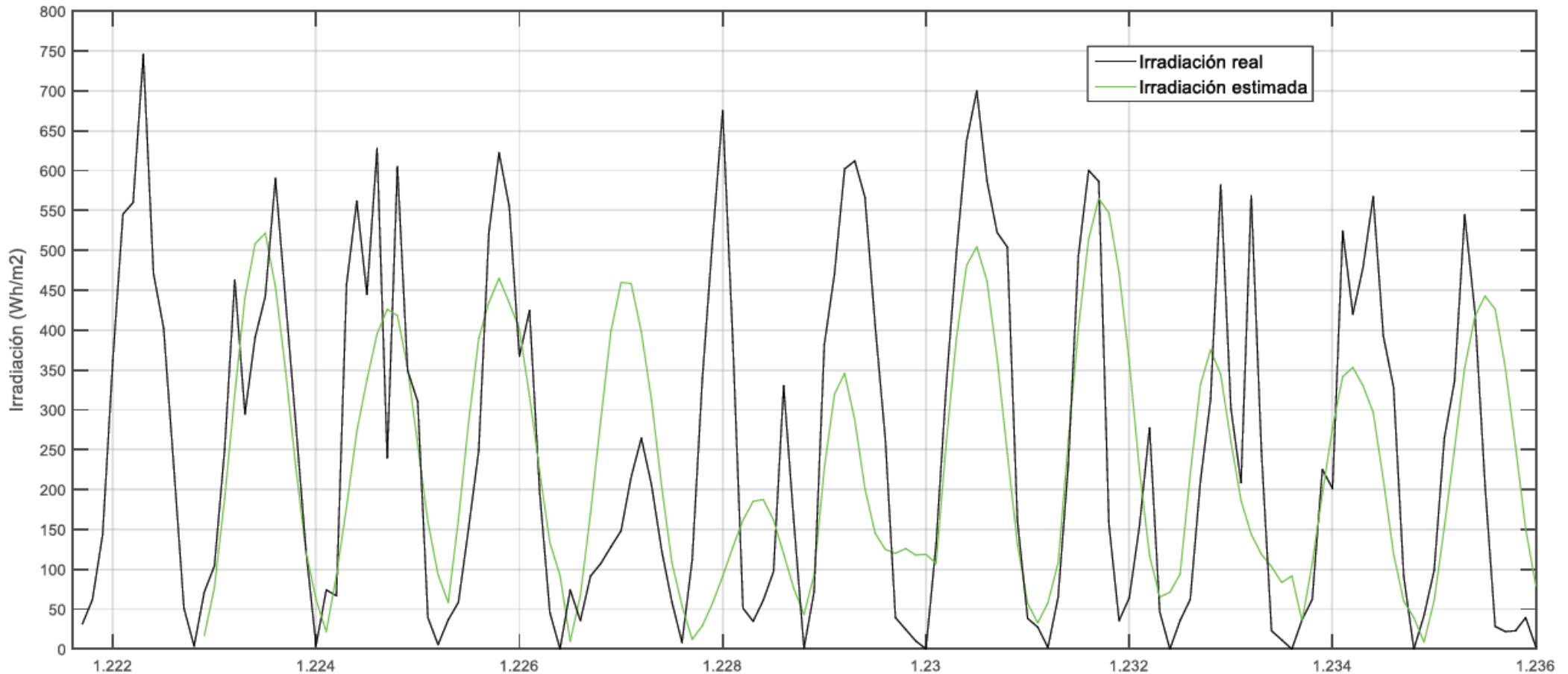
# Case Study



One hour ahead forecast, AR model,  $p=12$ , FIT=44%.



# Case Study



12 hours ahead forecast, AR model,  $p=12$ , FIT=24%.

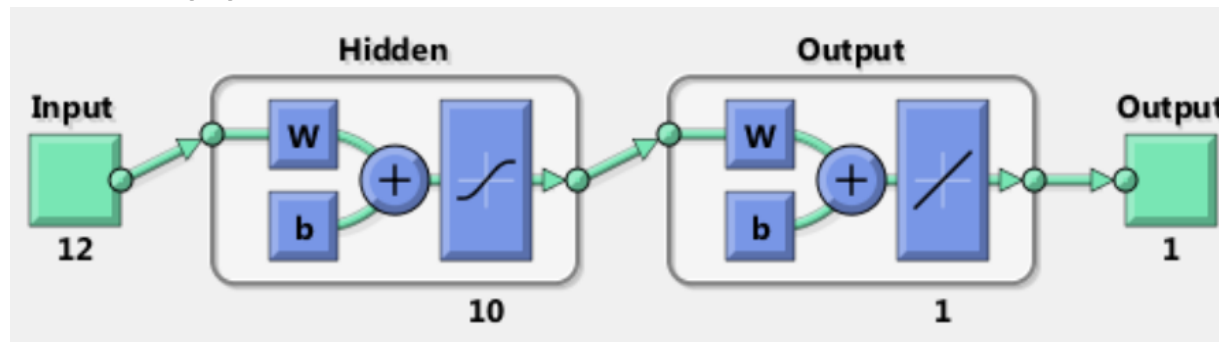


# Case Study

- Linear estimators are optimal for joint Gaussian distributions.
- In general, the conditional mean is a non-linear map, from available information to the optimal estimate.
- It can be approximated as a non-linear function from data:

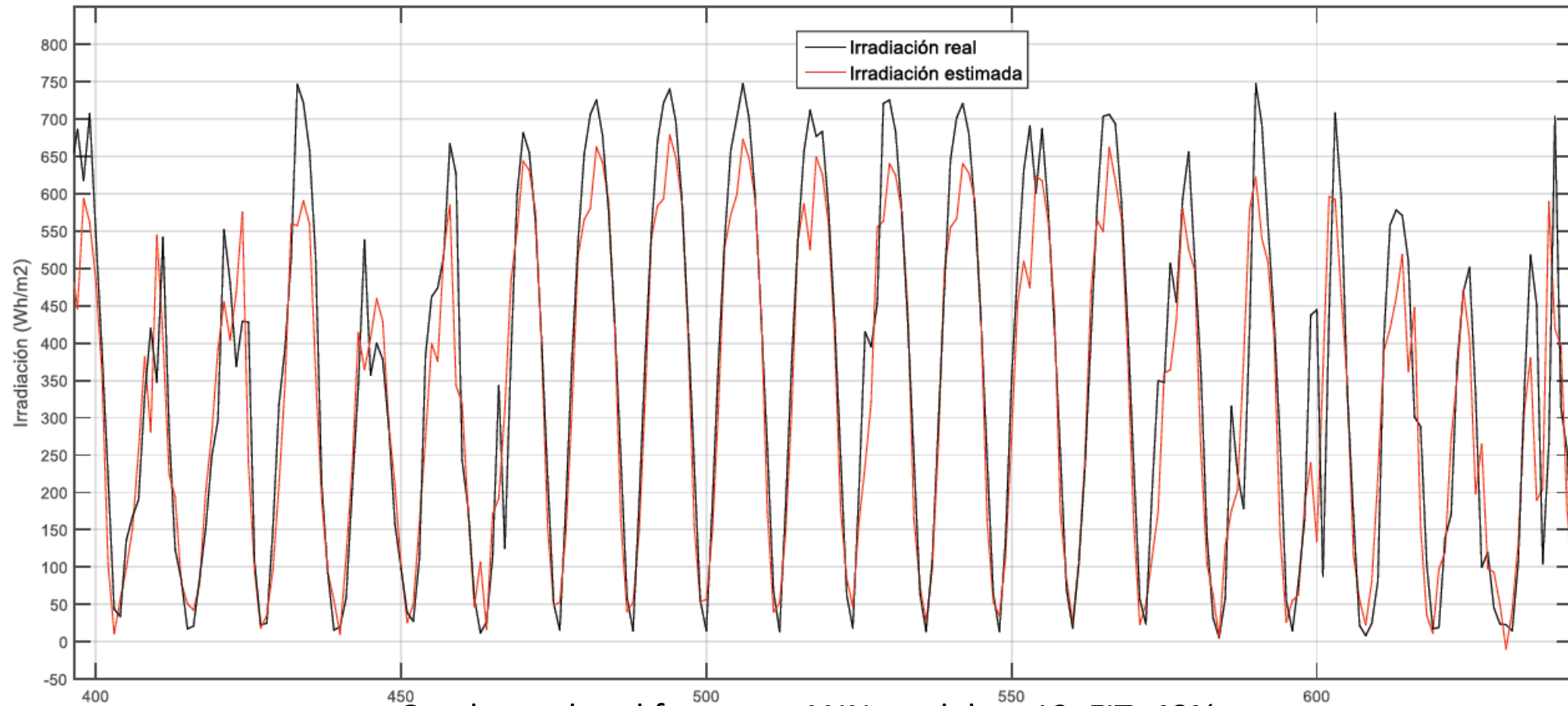
$$\hat{y}(t) = \mathbf{G}(y(t-1), y(t-2), \dots, y(t-p))$$

- *ANN are universal approximators:*





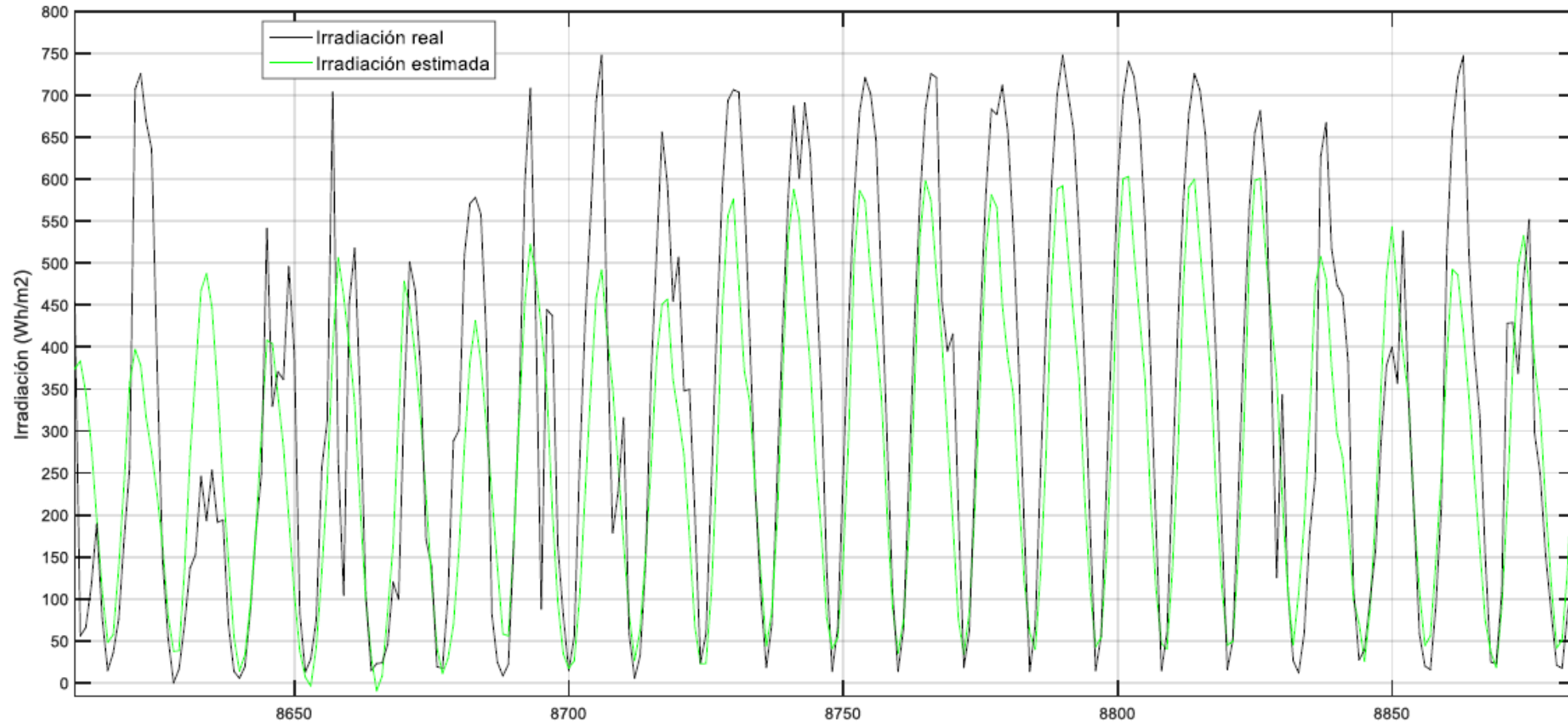
# Case Study



One hour ahead forecast, ANN model,  $p=12$ , FIT=48%.



# Case Study



12 hours ahead forecast, ANN model,  $p=12$ , FIT=32%.



# Case Study

Model	Horizon	FIT (%)	RMSE (Wh/m <sup>2</sup> )
AR	1	44	114
ANN	1	48	105
AR	12	24	162
ANN	12	32	144



# Bibliography

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