Lezione 27/02/2004
Testo pagg. 77-80

1) Cinematica inversa di posizione: quando sono possibili soluzioni in forma chiusa
   a) Esempi 2 & 3 gradi di movimento planari
2) Polso sferico
3) Manipolabilità all’interno dello spazio di lavoro, ellisse di manipolabilità
4) Cinematica di velocità
   a) Richiamo sulle velocità lineari e angolari
   b) Esempio di manipolatore a 2 gradi di movimento nello spazio
   c) Velocità lineare
   d) Matrice jacobiana

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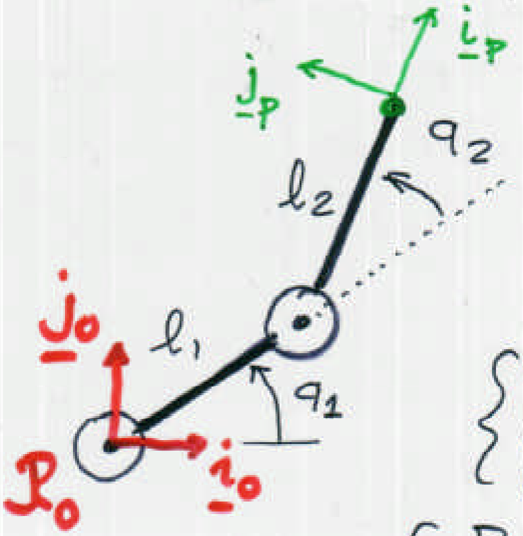
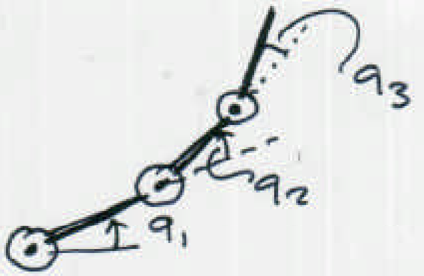


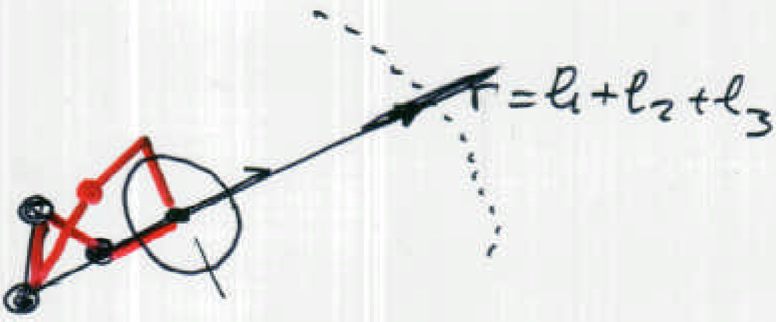
$$\begin{cases} x = l_1 \cos q_1 + l_2 \cos(q_1+q_2) \\ y = l_1 \sin q_1 + l_2 \sin(q_1+q_2) \\ \alpha = q_1 + q_2 \end{cases}$$

C.D.P

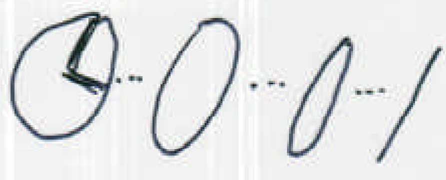
$$q_1 = f_1(x,y,\alpha)$$
$$q_2 = f_2(x,y,\alpha)$$

$$x - l_2 \cos\alpha = l_1 \cos q_1$$
$$y - l_2 \sin\alpha = l_1 \sin q_1$$





$$\underline{x}^T \underset{2\times2}{M} \underline{x}$$



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$\underline{p} = \underline{f}(\underline{q})$ FACILE

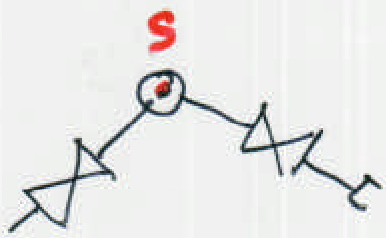
$\underline{q} = \underline{f}^{-1}(\underline{p})$ DIFFICILE

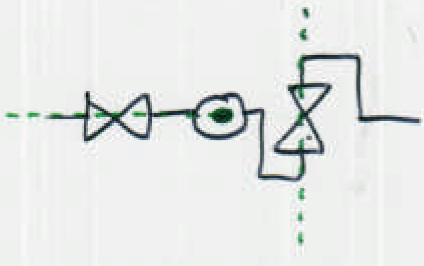
SE IL ROBOT HA

POLSO SFERICO

LA CIN. INV. POS. ESISTE

POLSO SFERICO = I TRE ASSI DI ROTAZ. DEI GIUNTI DEL POLSO SI INTERSECANO SEMPRE IN UN PUNTO **S**





CIN. INV. POS. per via NUMERICA

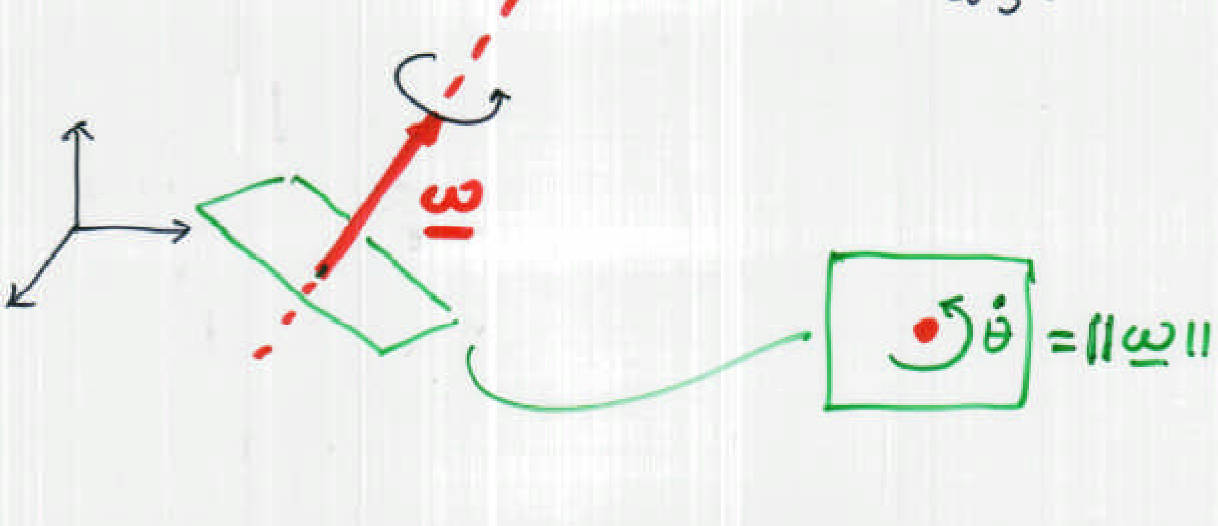
$$\underline{q} - \underline{f}^{-1}(\underline{p}) \simeq 0$$

CIN. VELOCITÀ

derivando la CIN. POSIZ

DIRETTA: $\underline{\dot{p}} = \underline{g}(\underline{\dot{q}})$

$$\dot{q} = \begin{bmatrix} \dot{q}_1 \\ \vdots \\ \dot{q}_6 \end{bmatrix} \qquad \underline{\dot{p}} = \begin{bmatrix} \frac{d}{dt}\underline{x} \\ \frac{d}{dt}\underline{\alpha} \end{bmatrix} = \begin{matrix} \left.\begin{matrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{matrix}\right\} \underline{v}_{PO} \\ \left.\begin{matrix} \dot{\alpha}_1 \\ \dot{\alpha}_2 \\ \dot{\alpha}_3 \end{matrix}\right\} \underline{\omega} \end{matrix}$$



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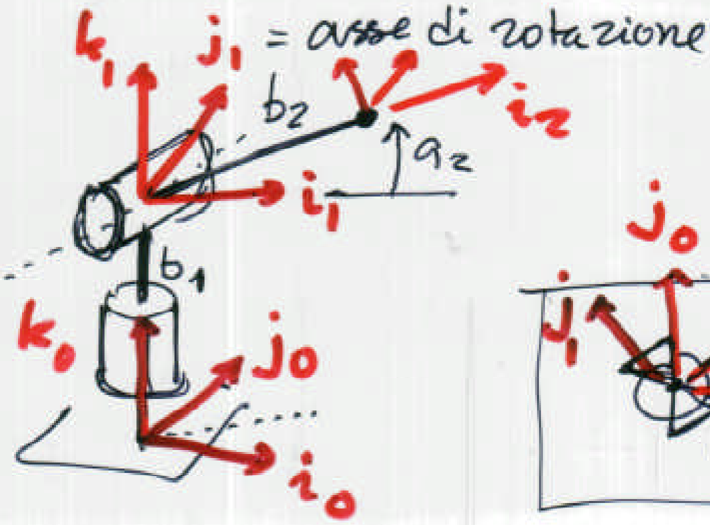

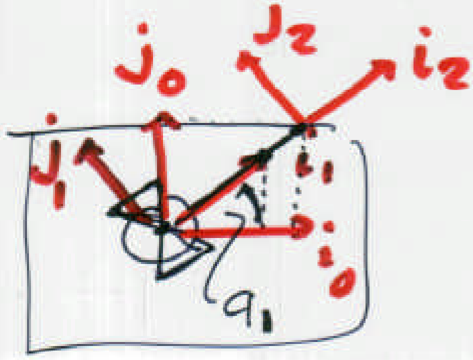

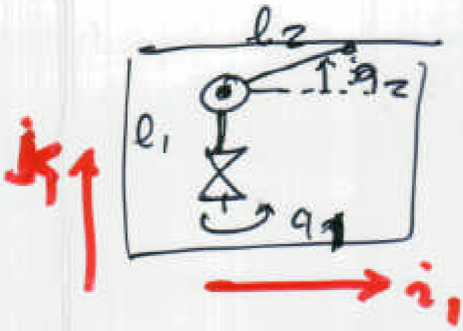

$$\begin{bmatrix} x \\ y \\ z \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} =$$

$x_1 = l_2 \cos q_2 \longrightarrow x_0 = l_2 \cos q_1 \cos q_2$

$z_1 = l_2 \sin q_2 \longrightarrow z_0 = l_1 + l_2 \sin q_2$

$y_1 = 0 \longrightarrow y_0 = l_2 \sin q_1 \cos q_2$

$$\underline{x} = \begin{bmatrix} l_2 c_1 c_2 \\ l_2 s_1 c_2 \\ l_1 + l_2 s_2 \end{bmatrix} \begin{matrix} f_1 \\ f_2 \\ f_3 \end{matrix} \rightarrow \underline{\dot{x}} = \frac{d}{dt}\underline{x} = \begin{bmatrix} ? \end{bmatrix}$$

$\underline{\alpha} = ?$

$$\frac{d}{dt}\{\ell_2 c_1 c_2) = \ell_2 \left(c_1(-s_2)\dot{q}_2 + c_2(-s_1)\dot{q}_1\right)$$

$$= -\ell_2\left[c_1 s_2 \dot{q}_2 + c_2 s_1 \dot{q}_1\right]$$

$$= \begin{bmatrix} -\ell_2 c_2 s_1 & -\ell_2 c_1 s_2 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

$$\frac{d}{dt}\left[\ell_2 s_1 c_2\right] = \ell_2\left[s_1(-s_2)\dot{q}_2 + c_2 c_1 \dot{q}_1\right]$$

$$= \begin{bmatrix} \ell_2 c_1 c_2 & -\ell_2 s_1 s_2 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

$$\frac{d}{dt}\left[\ell_1 + \ell_2 s_2\right] = \ell_2 c_2 \dot{q}_2 = \begin{bmatrix} 0 & \ell_2 c_2 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

$$\dot{\underline{x}} =$$

3×2

$$\text{Jacobiano} = \begin{bmatrix} \frac{\partial f_1}{\partial q_1} & \frac{\partial f_1}{\partial q_2} \\ \frac{\partial f_2}{\partial q_1} & \frac{\partial f_2}{\partial q_2} \\ \frac{\partial f_3}{\partial q_1} & \frac{\partial f_3}{\partial q_2} \end{bmatrix}$$

$$\underline{\dot{x}} = J_L \underline{\dot{q}} = J_L(\underline{q})\,\underline{\dot{q}}$$

LINEARE, MA
DIPENDENTE DALLA
CONFIGURAZIONE ISTANTANEA
DEL MANIPOLATORE