Lezione 04/03/2004
Testo pagg. 50 e segg.

a) Rototraslazioni
b) Angoli di Eulero
c) Angoli RPY

ROTOTRASLAZIONI

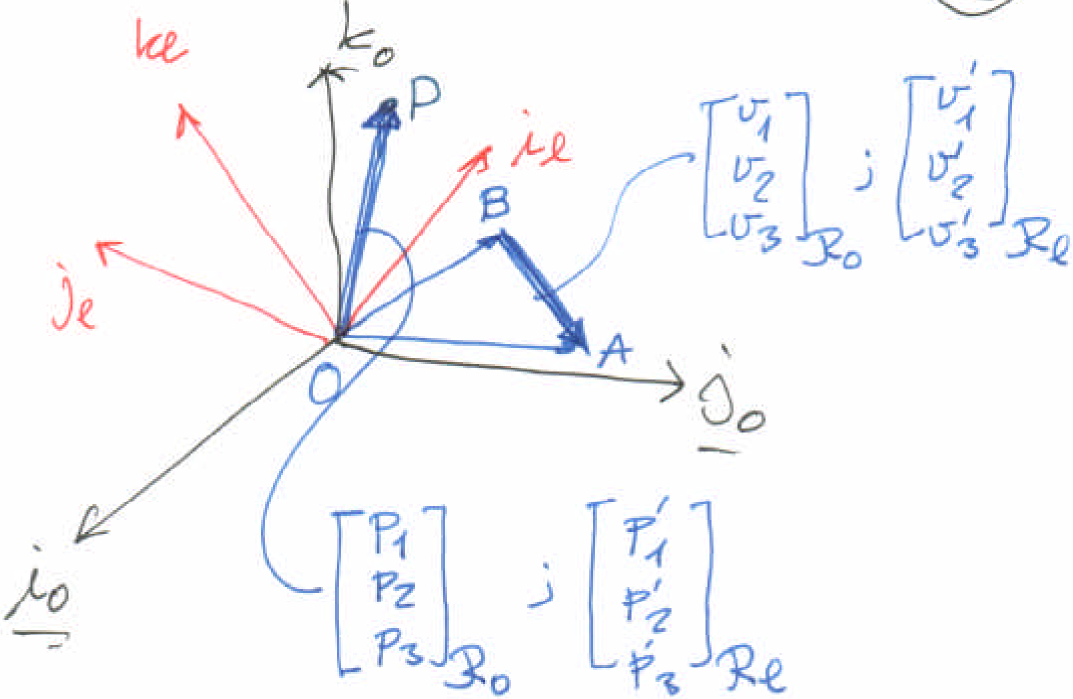
$\underbrace{\text{TO}}_{\text{NO}}$

R_e^o matrice 3x3 ortonormale; $\det R_e^o = +1$

1) R_e^o rappresenta la rotazione rigida che porta $\mathcal{R}_o$ in $\mathcal{R}_e$

2) le colonne di R_e^o rappresentano i versori di $\mathcal{R}_e$ in $\mathcal{R}_o$

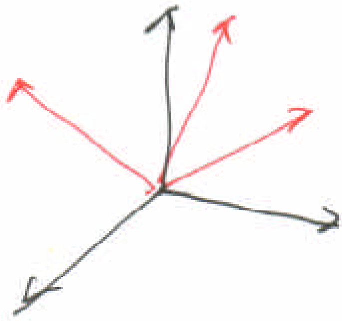
3) R_e^o rappresenta l'operatore (lineare) che trasforma un vettore rappresentato in $\mathcal{R}_e$ nello stesso vettore rappresentato in $\mathcal{R}_o$



$\underline{v} = \underline{A} - \underline{B}$ $\qquad$ $\underline{p} = \underline{P} - \underline{O}$

$$[\underline{p}]_{\mathcal{R}_0} = R_e^0 [\underline{p}]_{\mathcal{R}_e}$$

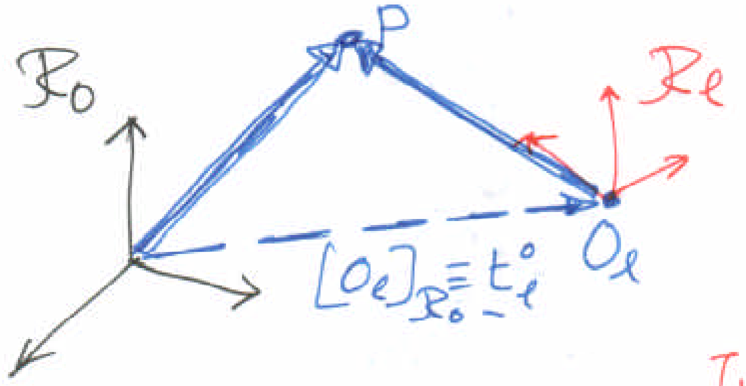
$$[\underline{v}]_{\mathcal{R}_0} = R_e^0 [\underline{v}]_{\mathcal{R}_e}$$



Rappresentiamo $\underline{i}_\ell , \underline{j}_\ell , \underline{k}_\ell$ in $\mathcal{R}_0$

$$\underline{i}_0 = R^0_\ell \, \underline{i}_\ell = \begin{bmatrix} \boxed{} & \cdots & \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \boxed{}$$

$$\underline{j}_0 = R^0_\ell \, \underline{j}_\ell = \begin{bmatrix} \cdots & \boxed{} & \cdots \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \boxed{}$$



$$[\underline{p}]_{\mathcal{R}_\ell} = [P]_{\mathcal{R}_\ell} \boxed{- [O_\ell]_{\mathcal{R}_\ell}}$$

$[t_\ell^0]_{\mathcal{R}_0}$

$$\Downarrow$$

$$[\underline{p}]_{\mathcal{R}_0} = R_\ell^0 [P]_{\mathcal{R}_\ell} - \cancel{R_\ell^0 [O_\ell]_{\mathcal{R}_\ell}}$$

troppo schematico

NON VA BENE

A NOI INTERESSA $[O_\ell]_{\mathcal{R}_0}$

$$[\underline{p}]_{\mathcal{R}_0} = R_\ell^0 [\underline{p}]_{\mathcal{R}_\ell} + \underline{t}_\ell^0$$

$$[\underline{v}]_{\mathcal{R}_0} = R_\ell^0 [A]_{\mathcal{R}_\ell} + \underbrace{\cancel{t_\ell^0} - R_\ell^0 [B]_{\mathcal{R}_\ell}}_{A-B} - \cancel{t_\ell^0}$$

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ROTOTRASLAZIONI : COME OPERIAMO?

USANDO LE COORDINATE OMOGENEE E LE MATRICI DI TRASF. OMOGENEA

$[\underline{p}]_{\mathcal{R}_\ell}$ dato

$\Downarrow$

vettore omogeneo

$$[\tilde{p}]_{\mathcal{R}_\ell} = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \\ 1 \end{pmatrix}_{\mathcal{R}_\ell}$$

$\Downarrow$

$$[\tilde{p}]_{\mathcal{R}_0} = \begin{bmatrix} \mathcal{R}^0_\ell & \underline{t}^0_\ell \\ \underline{0}^T & 1 \end{bmatrix} [\tilde{p}]_{\mathcal{R}_\ell}$$

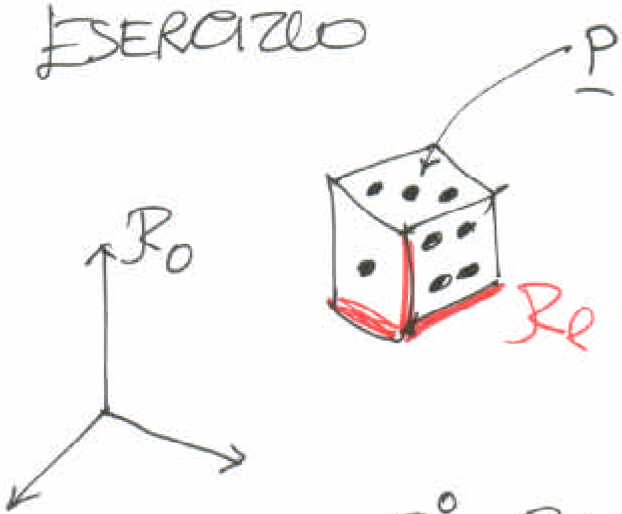
4x4

$\underline{0}^T$ = [0 0 0]

$\Downarrow$

$[p]_{\mathcal{R}_0}$ togliendo l'1 finale

ESERCIZIO



$$[P]_{\mathcal{R}_l} = \begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$R_l^0 = Rot(\underline{k}, 90°)\, Rot(\underline{j}, -45)$$

$$t_l^0 = \begin{bmatrix}1\\1\\0\end{bmatrix}$$

$$R_l^0 = \begin{bmatrix}0 & -1 & 0\\1 & 0 & 0\\0 & 0 & 1\end{bmatrix} \cdot \begin{bmatrix}\sqrt{2}/2 & 0 & -\sqrt{2}/2\\0 & 1 & 0\\\sqrt{2}/2 & 0 & \sqrt{2}/2\end{bmatrix} =$$

$$= \begin{bmatrix}0 & -1 & 0\\\sqrt{2}/2 & 0 & -\sqrt{2}/2\\\sqrt{2}/2 & 0 & \sqrt{2}/2\end{bmatrix}$$

$$T_l^0 = \begin{bmatrix} R_l^0 & \begin{bmatrix}1\\1\\0\end{bmatrix} \\ \underline{0}^T & 1\end{bmatrix}$$

$$T_e^0 \begin{bmatrix} 1 \\ 1 \\ 1 \\ \cdots \\ 1 \end{bmatrix}$$

← strutturale : è un vettore omogeneo !

$$\left[\begin{array}{ccc|c} 0 & -1 & 0 & 1 \\ \sqrt{2}/2 & 0 & -\sqrt{2}/2 & 1 \\ \sqrt{2}/2 & 0 & \sqrt{2}/2 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \begin{bmatrix} 1 \\ 1 \\ 1 \\ \hline 1 \end{bmatrix} =$$

$$= \begin{bmatrix} -1+1 \\ \sqrt{2}/2 - \sqrt{2}/2 + 1 \\ \sqrt{2}/2 + \sqrt{2}/2 \\ \hline 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ \sqrt{2} \\ 1 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}_{\mathcal{R}_e} \Rightarrow \begin{bmatrix} 0 \\ 1 \\ \sqrt{2} \end{bmatrix}_{\mathcal{R}_0}$$

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$$R_e^0 = \begin{bmatrix} [\ ] & [\ ] & [\ ] \end{bmatrix}$$

9 parametri (e 6 vincoli)

estrarre $\Downarrow$

3 parametri angolari: come li estraiamo

ANGOLI DI EULERO $\{\phi, \theta, \psi\}$

$$R_{\phi,\theta,\psi} = Rot(\underline{k}, \phi)\, Rot(\underline{i}, \theta)\, Rot(\underline{k}, \psi)$$

$Rot(j, \theta)$

ZXZ "mobile" 50%

ZYZ "mobile" 50%

ANGOLI (DI) RPY $\{\theta_1, \theta_2, \theta_3\}$

$$R_{\theta_1,\theta_2,\theta_3} = Rot(\underline{k}, \theta_3)\, Rot(j, \theta_2)\, Rot(\underline{i}, \theta_1)$$

XYZ "fisso"

$$Rot(\underline{k}, \theta) = \begin{bmatrix} c_\theta & -s_\theta & 0 \\ s_\theta & c_\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$