Lezione 16/03/2004

a) Proprietà delle matrici nelle equazioni dinamiche del manipolatore: testo pagg. 120-123
b) Passaggio alla rappresentazione in variabili di stato: testo pagg. 123-125

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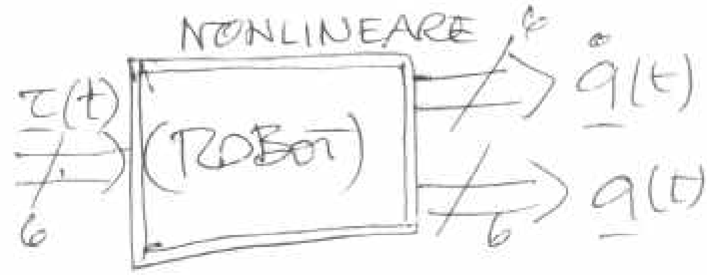
IPOTESI:

a) ASSUME RIGIDITÀ PERFETTA

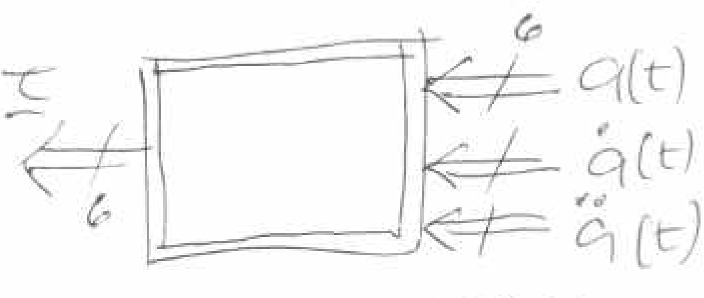
b) " ASSENZA DI ATTRITI

$$H(\underline{q})\ddot{\underline{q}}(t) + C(\underline{q},\dot{\underline{q}})\dot{\underline{q}}(t) + \underline{g}(\underline{q}) = \underline{\tau}$$

n (=6) equazioni (ROBOT)



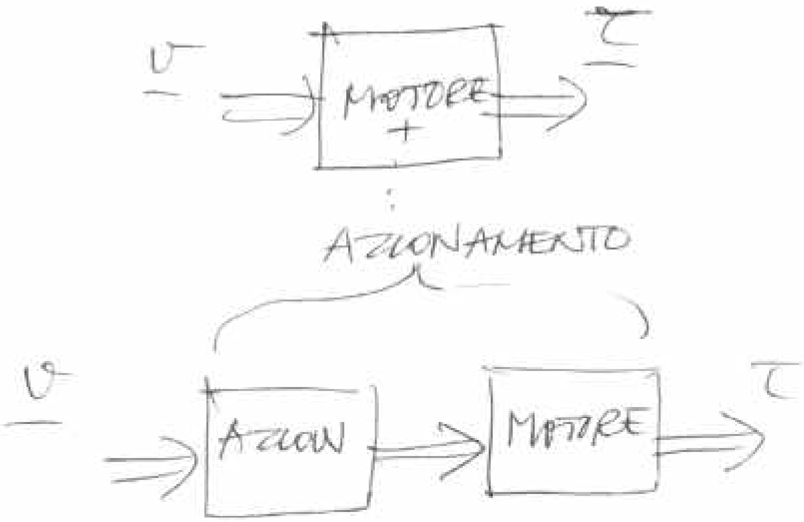
DINAMICA DIRETTA



DINAMICA INVERSA

COME GENERIAMO $\underline{\tau}$?

TECNOLOGICAMENTE



$H(\underline{q})$ è sempre invertibile!

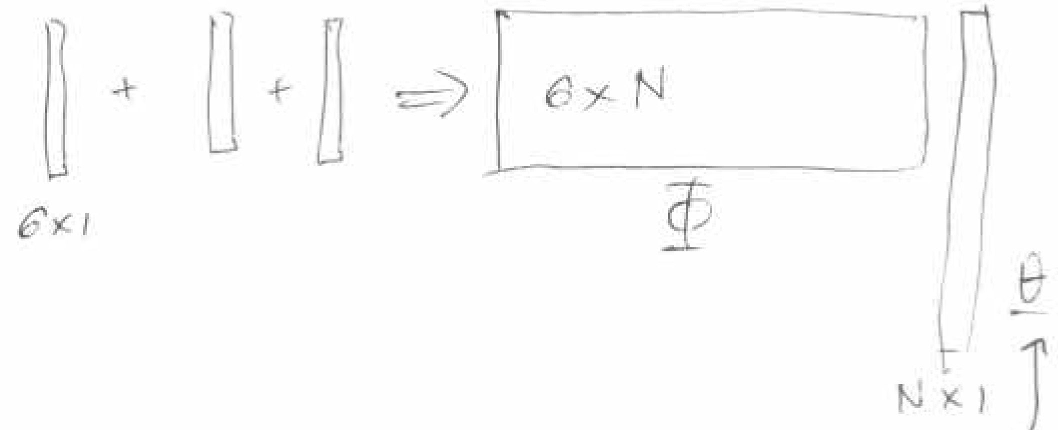
$H(\underline{q}) > 0$ definita positiva

$$\dot{H}(\underline{q}) - 2C(\underline{q}, \underline{\dot{q}}) = N(\underline{q}, \underline{\dot{q}})$$

$$\underbrace{\underline{\dot{q}}^T N(\underline{q}, \underline{\dot{q}}) \underline{\dot{q}}}_{\text{FORMA QUADRATICA ASSOCIATA}} \equiv 0$$

$= 0$

$$H\underline{\ddot{q}} + C\underline{\dot{q}} + \underline{g} \Rightarrow \underline{\Phi}(\underline{\ddot{q}}, \dot{q}, q)\,\underline{\theta}$$



vettore dei parametri (geometrici e o fisici)

Forma di regressione

$$H(\underline{q})\,\underline{\ddot{q}}(t) + C(\underline{q}, \underline{\dot{q}})\,\underline{\dot{q}}(t) + \underline{g}(\underline{q}) = \underline{\tau}(t)$$

$\Downarrow$ VARIABILI DI STATO

$$\underset{12\times 1}{\underline{x}(t)} = \begin{bmatrix} \underline{x}_1(t) \\ \underline{x}_2(t) \end{bmatrix} = \begin{bmatrix} \underline{q}(t) \\ \underline{\dot{q}}(t) \end{bmatrix} \begin{array}{l} 6\times 1 \\ 6\times 1 \end{array}$$

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$$\dot{\underline{x}}_1(t) = x_2(t)$$

$$\dot{\underline{x}}_2(t) = \ddot{q}(t) = H^{-1}(\underline{q})\left[\underline{\tau} - \underline{C}\dot{\underline{q}} - \underline{g}\right]$$

$$= H^{-1}(\underline{x}_1(t))\left[-C(\underline{x}_1, \underline{x}_2)\underline{x}_2(t) - \right.$$
$$\left. - \underline{g}(x_1(t)) + \tau(t)\right]$$

EQUAZIONE IN VARIABILI DI STATO

$$\dot{\underline{x}}(t) = A(\underline{x}(t)) + B(\underline{x}(t))\,\underline{u}(t)$$

$$\dot{x}(t) = Ax(t) + Bu(t)$$

LINEARE
CONTINUO
INVARIANTE NEL TEMPO

$$A(\underline{x}(t)) = \begin{bmatrix} \underline{x}_2(t) \\ -H^{-1}(x_1(t))\left[C(\underline{x}_1(t), \underline{x}_2(t)) + \underline{g}(\underline{x}_1(t))\right] \end{bmatrix}$$

$$B(\underline{x}(t)) = \begin{bmatrix} 0 \\ H^{-1}(x_1(t)) \end{bmatrix}$$

$$\underline{u}(t) = \underline{\tau}(t)$$

APPROSSIMAZIONE SUL MODELLO
X PROGETTARE IL CONTROLLO

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(A)

$$H(\underline{q}) = H = \text{costante} = \begin{bmatrix} h_{11} & & 0 \\ & h_{22} & \\ 0 & & \ddots \end{bmatrix}$$

(B)

$$C(q,\dot q)\dot q \simeq \underline{0}$$

trascuriamo i termini centrifughi e di Coriolis

$$H\ddot{\underline{q}} + \underline{g}(\underline{q}) = \underline{\tau}$$

$$\Downarrow$$

$$\begin{cases} \dot x_1(t) = x_2(t) \\ \dot x_2(t) = H^{-1}\underline{\tau} - H^{-1}g(\underline{q}) \end{cases}$$

$$\underline{\dot x} = [\;] = [\,A\,]\,\overset{\underline{x}}{[\;]} + B\,[\underline{u}] + \underline{W}$$

$$\Downarrow$$

$$\begin{bmatrix} \dot x_1 \\ \dot x_2 \end{bmatrix} = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ H^{-1} \end{bmatrix}\underline{\tau} + W$$

$W = H^{-1}g(q)$

DISTURBO X GRAVITÀ

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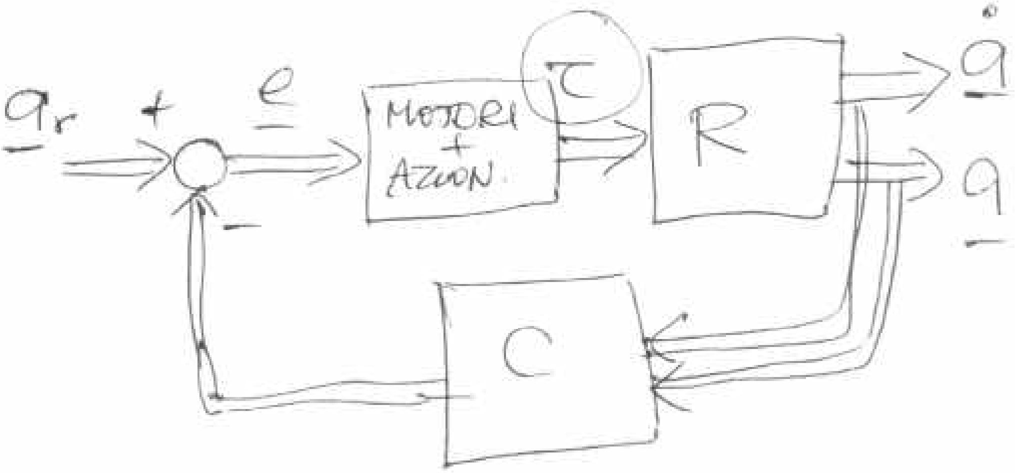
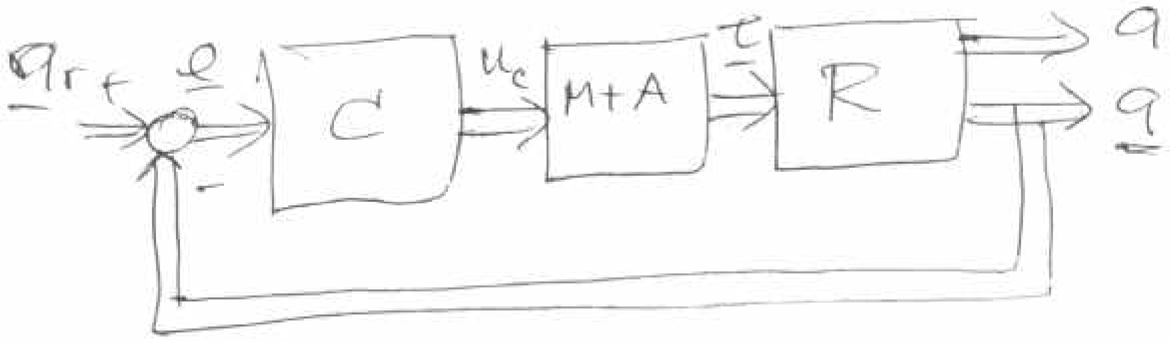
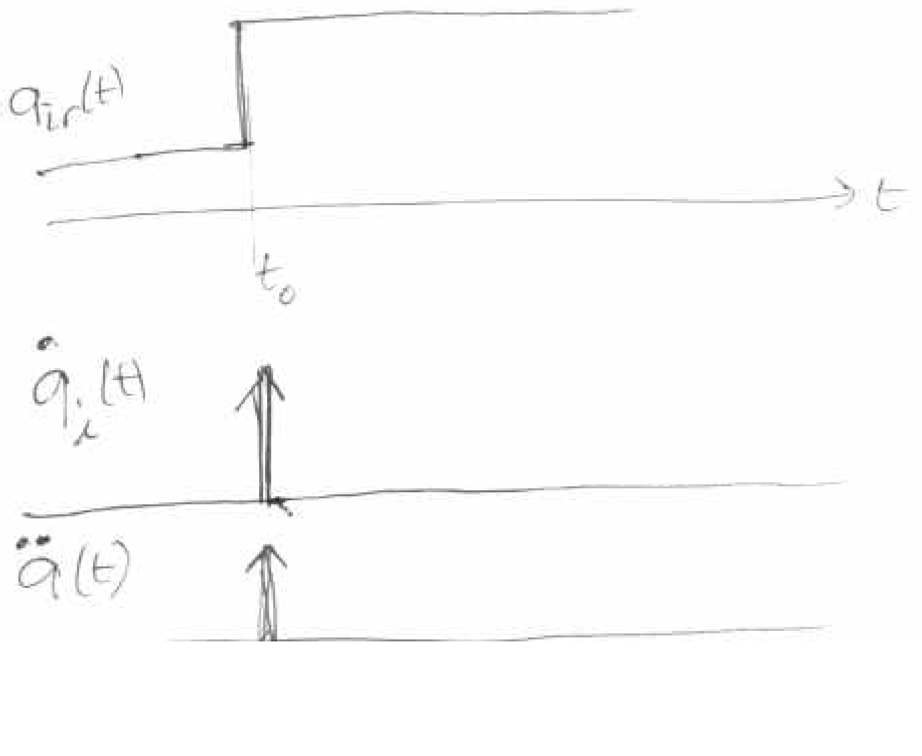


$\underline{q}_r$
+
$\underline{e}$
−
MOTORI + AZION.
τ
R
$\dot{\underline{q}}$
$\underline{q}$
C

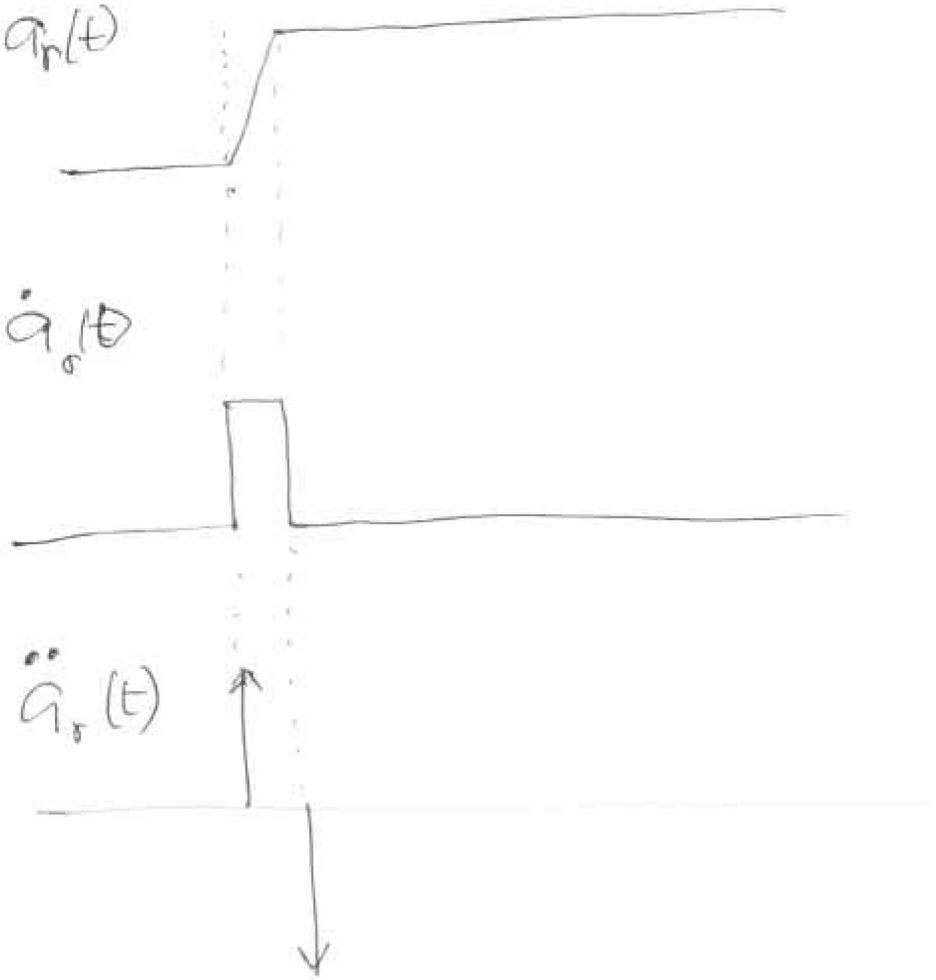


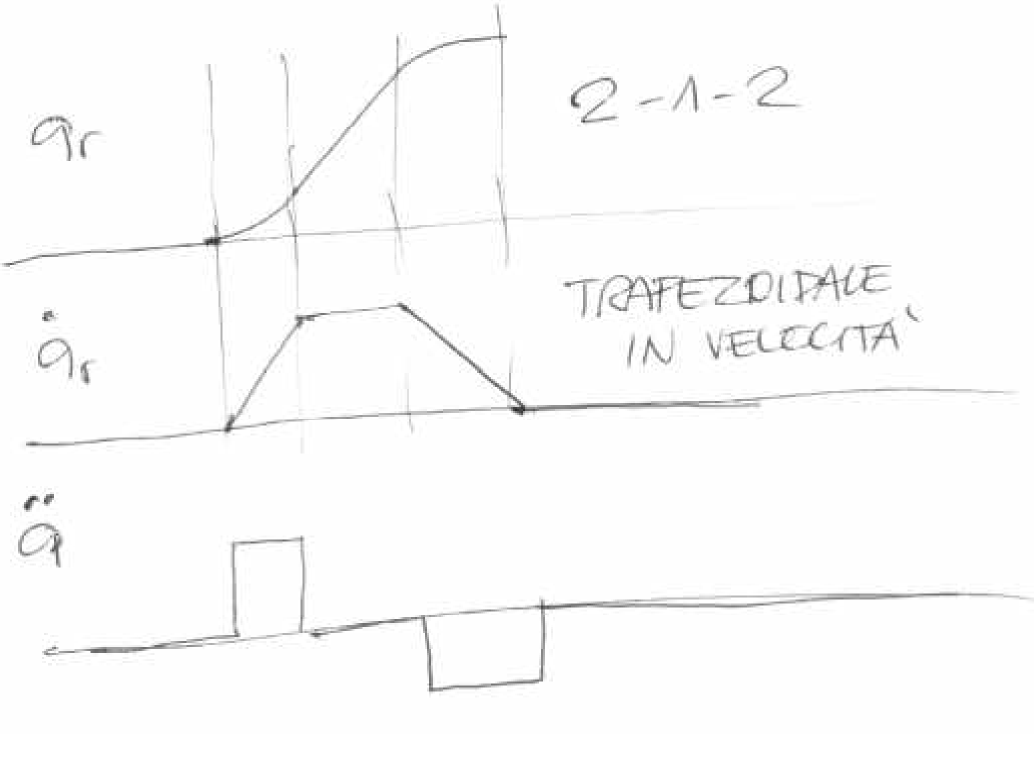
$\underline{q}_r$
+
$\underline{e}$
−
C
u_c
M+A
$\underline{\tau}$
R
$\dot{q}$
$\underline{q}$



$q_{ir}(t)$
t
t_0
$\dot{q}_i(t)$
$\ddot{q}(t)$

7


$q_r(t)$
$\dot{q}_r(t)$
$\ddot{q}_r(t)$


q_r
2-1-2
$\dot{q}_r$
TRAPEZOIDALE IN VELOCITÀ
$\ddot{q}$