CONTROLLO A DINAMICA INVERSA

Modello Robot

$$M(\underline{q})\,\underline{\ddot{q}}(t) + \underline{h}(\underline{q}, \underline{\dot{q}}) = \underline{u}_c$$

Controllo così fatto

$$\underline{u}_c := \hat{M}(\underline{\ddot{q}}_r - \underline{v}_c) + h$$

$$\ddot{q}_r - \ddot{q} = v_c - \eta(q, \dot{q}, \ddot{q}_r, v_c)$$

$$\eta = [M^{-1}(q)\hat{M} - I](\ddot{q}_r - v_c) - M^{-1}(q)\,\Delta h(q, \dot{q})$$

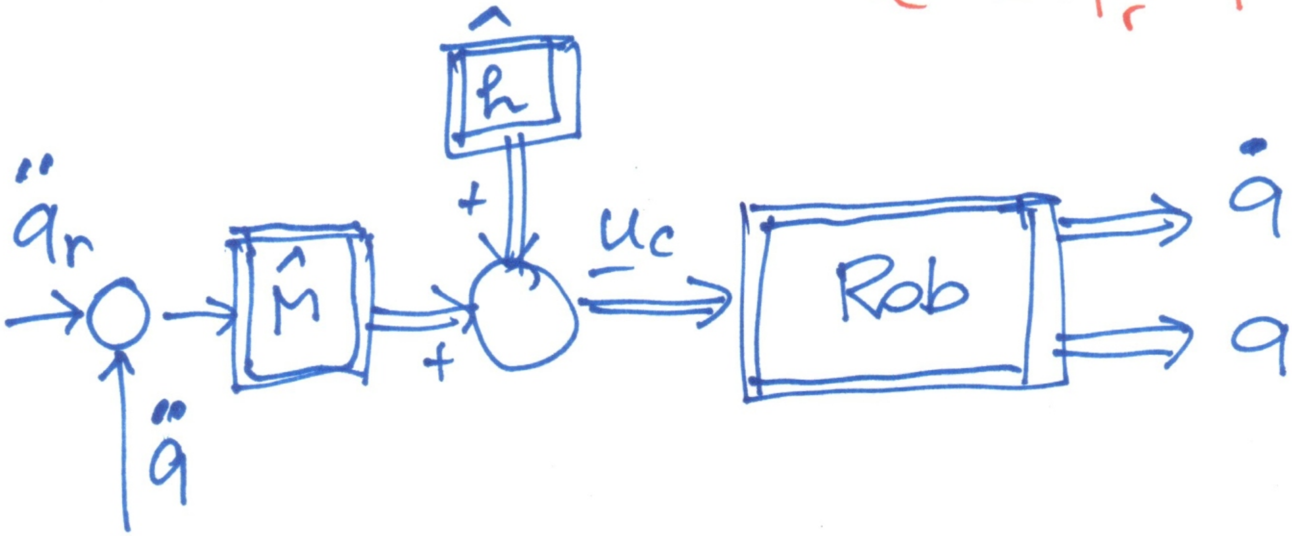
$$\underline{\ddot{e}} = \underline{v}_c - \eta$$

$$e := q_r - q$$

$$\dot{e} := \dot{q}_r - \dot{q}$$

$$\ddot{e} = \ddot{q}_r - \ddot{q}$$

$$s^2 e = v_c - \eta$$



$$\underline{x}(t) = \begin{bmatrix} \underline{x}_1(t) \\ \underline{x}_2(t) \end{bmatrix} = \begin{bmatrix} \underline{q}(t) \\ \underline{\dot{q}}(t) \end{bmatrix}$$

$$\underline{\dot{x}}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} \dot{q}(t) \\ \ddot{q}(t) \end{bmatrix} = A\underline{x} + B(\underline{\ddot{q}}_r - \underline{v}_c) + B\underline{\eta}(\cdot)$$

$$A = \begin{bmatrix} 0 & I_6 \\ 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ I_6 \end{bmatrix}$$

$$\underline{\varepsilon}(t) = \begin{bmatrix} \varepsilon_1(t) \\ \varepsilon_2(t) \end{bmatrix} = \begin{bmatrix} e(t) \\ \dot{e}(t) \end{bmatrix} = \begin{bmatrix} q_r(t) - q(t) \\ \dot{q}_r(t) - \dot{q}(t) \end{bmatrix}$$

$$\underline{\dot{\varepsilon}}(t) = A\underline{\varepsilon} + B\underline{v}_c - B\eta(\cdots) \qquad q, \dot{q} \quad \ddot{q}_r \quad v_c$$

C.C. & C.O.

COME PROGETTARE $\underline{v}_c$? USCITA DI UN CONTROLLORE (SISTEMA) DINAMICO

$$\begin{cases} \dot{\xi}(t) = F\xi(t) + G\varepsilon(t) \\ \underline{v}_c = H\xi(t) + L\varepsilon(t) \end{cases}$$

$$+$$

$$\underline{\dot{\varepsilon}} = A\underline{\varepsilon} + B\underline{v}_c - B\eta(\cdots)$$

DOPPIO ANELLO

- INNER LOOP : NONLINEARE e SERVE A LINEARIZZARE GLOBALMENTE

$\underline{u}_c := \dots$

- OUTER LOOP LINEARE e SERVE a STABILIZZARE + PRESTAZIONI

$v_c := \cdots$

LINEARIZZ. ESATTA

$$\left.\begin{aligned}\hat{M} &:= M(q)\\ \hat{h} &:= h(q,\dot{q})\end{aligned}\right\} \longrightarrow \begin{aligned}&E(q) = 0\\ &\eta(\cdots) \equiv 0\\ &\forall q, \dot{q}, \ddot{q}_r, v_c, t\end{aligned}$$

$$\boxed{\ddot{q}_r - \ddot{q} \equiv \ddot{e} = v_c}$$

$v_c(t)$?

RETE PD $\quad v_c := -K_D\dot{e} - K_P e = \overbrace{\begin{bmatrix}-K_P & -K_D\end{bmatrix}}^{-K} \underline{e}$

$F = G = H = 0 \qquad L = -K$

$$\ddot{\underline{e}}(t) + K_D \dot{\underline{e}}(t) + K_P \underline{e}(t) = \underline{0}$$

K_D e K_P diagonali

i-esima eq

$$\ddot{e}_i + K_{Di} \dot{e}_i + K_{Pi} e_i = 0$$

$$(s^2 + K_{Di} s + K_P) e_i = 0 \qquad \frac{e_i}{0} = \frac{1}{s^2 + K_D s + K_P}$$

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases}$$

risposta all'ingresso u

$\longrightarrow C(sI - A)^{-1} B u = G(s) u$

risposta allo stato iniziale x_0

$\longrightarrow C(sI - A^{-1}) x_0$

PID

$$\underline{v}_c := -K_p \underline{e} - K_D \dot{\underline{e}} - K_I \int \underline{e}\, dt$$

$$\underline{v}_c = -\left(K_p + K_D s + K_I \frac{1}{s}\right) \underline{e}$$

$$\begin{cases} \dot{\xi} = G\varepsilon \\ v_c = H\xi + L\varepsilon \end{cases}$$

$$G = [I \quad 0]$$

$$H = -K_I$$

$$L = [-K_p \quad -K_D]$$

$$\ddot{e} + K_D \dot{e} + K_p e + K_I \int e\, dt = 0$$

$$\Downarrow \mathcal{L}$$

$$\left[I s^2 + K_D s + K_p + K_I \frac{1}{s}\right] e(s) = 0$$

$$P(s) = I s^3 + K_D s^2 + K_p s + K_I$$

$$\begin{bmatrix} \dot{\xi} \\ \dot{\varepsilon}_1 \\ \dot{\varepsilon}_2 \end{bmatrix} = \begin{bmatrix} 0 & I_6 & 0 \\ 0 & 0 & I_6 \\ -K_I & -K_p & -K_D \end{bmatrix} \begin{bmatrix} \xi \\ \varepsilon_1 \\ \varepsilon_2 \end{bmatrix}$$

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LINEARIZZ. APPROX.

ORE C'È

$$\dot{\varepsilon} = A\varepsilon + B\,(v_c - \eta)$$

DINAMICA INVERSA CON COMPENSAZ. ESATTA DELLA GRAVITÀ

$$\hat{M} = I$$

$$\hat{h} = g(q) - \ddot{q}_r$$

$$V(\underline{x}) = \frac{1}{2}\dot{q}^T M(q)\dot{q} + \frac{1}{2} e^T K_p e$$

$\dot{q}$ $\qquad$ $q_r - q$

$$\Downarrow$$

$$\dot{V}(\underline{x}) = -\dot{q}^T (K_D + B_t)\dot{q} \qquad \underline{x} = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$$