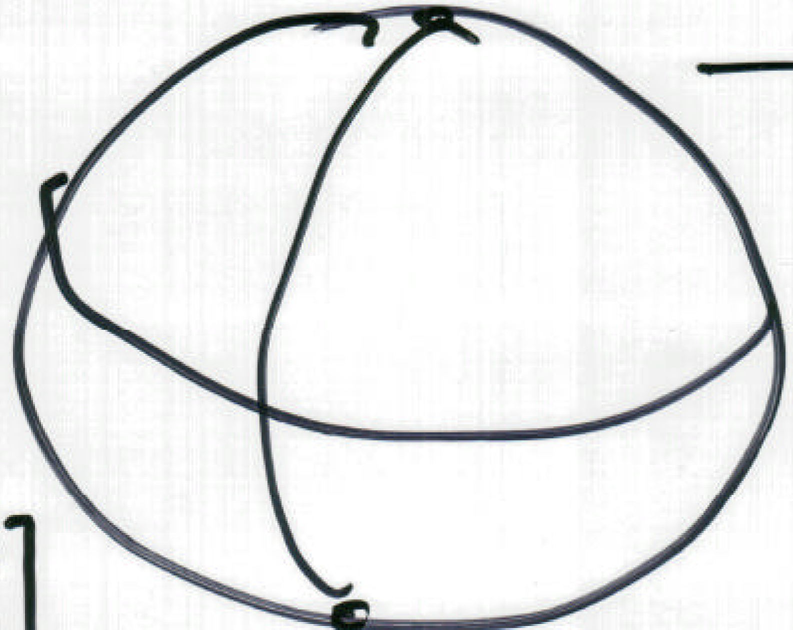
ROTO-TRASLAZIONI IN 3D

- CAP 2 TESTO
- APPENDICE
- APPUNTI MATRICI & VETTORI
- LEZIONI 2ª SETTIMANA 2004

GEOMETRIA
- PIANE
- 3D
- . . . .

→ EUCLIDEA



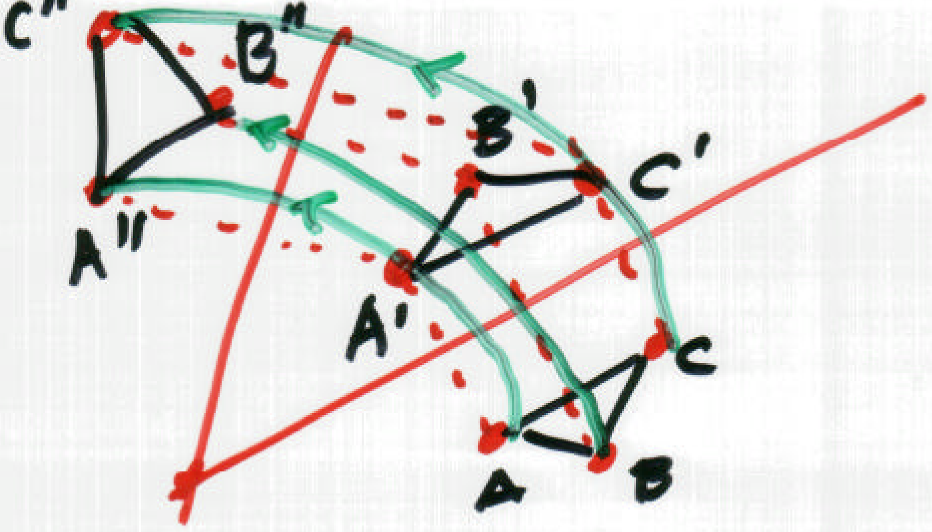
$P_1 \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}$ .... $P_2 \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}$

$$d(P_1 P_2) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

TRASFORMAZIONI ISOMETRICHE

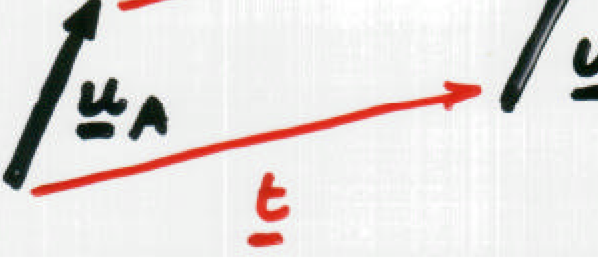
CONSERVAZIONE DELLE DISTANZE

- RIFLESSIONI → IN ROBOTICA NON SI USA
- ROTAZIONI
- TRASLAZIONI



ROTAZIONI + TRASLAZIONI

TRASLAZIONI ≡ SOMMA VETTORIALE

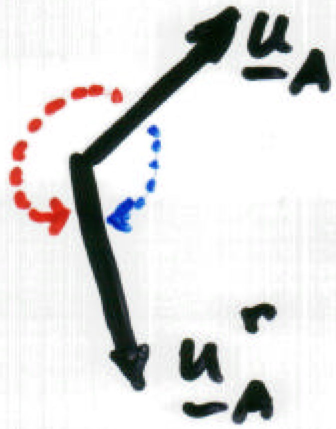


$$\underline{u}_A^t = \underline{u}_A + \underline{t}$$

$$\underline{u}_A = \underline{u}_A^t - t$$

ROTAZIONI

$$\underline{u}_A \rightsquigarrow ?\ \underline{u}_A^r$$



$$\underline{u}_A^r = R\,\underline{u}_A$$

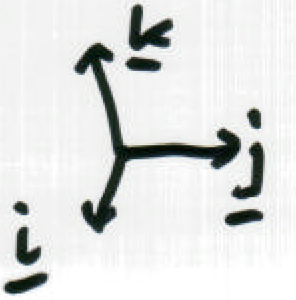
OPERATORE : PRODOTTO DI MATRICE

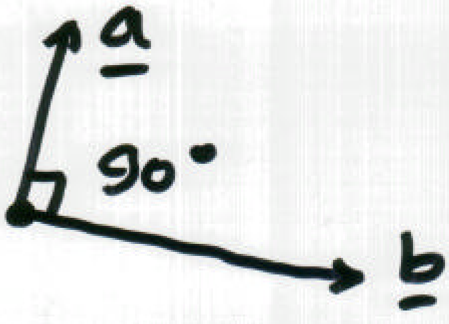
$$\underline{u}_A = R^{-1}\,\underline{u}_A^r$$

$$R^{-1} = R^T$$

R 3×3 di rotazione

- ORTO NORMALE righe/colonne ortogonali hanno norma 1 ortogonale





$\underline{a}^T \underline{b} = 0$ prodotto scalare è nullo

$\begin{bmatrix} \text{———————} \end{bmatrix} \begin{bmatrix} | \\ | \\ | \end{bmatrix} = 0$

$\underline{a} \cdot \underline{b} = \underline{a}^T \underline{b}$

$\underline{a} \times \underline{b}$ ⟵

$\underline{a} \wedge \underline{b}$

$$R = \begin{array}{c} \begin{bmatrix} | & | & | \\ | & | & | \\ | & | & | \end{bmatrix} \\ \underline{r}_1 \;\; \underline{r}_2 \;\; \underline{r}_3 \end{array}$$

$\underline{r}_i^T \underline{r}_i = 1$

$\underline{r}_i^T \underline{r}_j = 0$

$$R = \begin{bmatrix} - \; c_1 \; - \\ - \; c_2 \; - \\ - \; c_3 \; - \end{bmatrix}$$

$\underline{c}_i^T \underline{c}_i = 1$

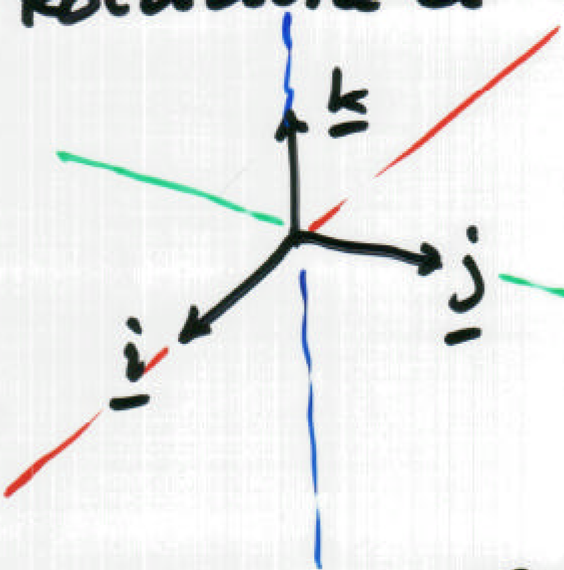
$\underline{c}_i^T \underline{c}_j = 0$

- $R^{-1} = R^T \qquad RR^T = R^TR = I$

- $\det(R) = +1$

Riflessioni $\bar{R}$ $\quad \det(\bar{R}) = -1$

Rotazione di α intorno a $\underline{i}$ (α)



$$\mathrm{Rot}(\underline{i}, \alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C_\alpha & -S_\alpha \\ 0 & S_\alpha & C_\alpha \end{bmatrix}$$

$$\mathrm{Rot}(\underline{j}, \beta) = \begin{bmatrix} C_\beta & 0 & S_\beta \\ 0 & 1 & 0 \\ -S_\beta & 0 & C_\beta \end{bmatrix}$$

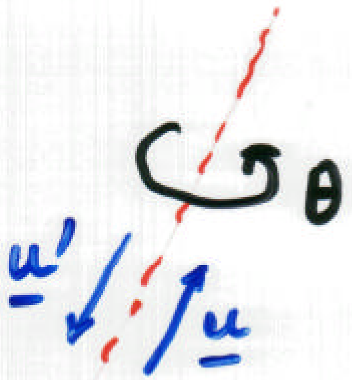
$$\mathrm{Rot}(\underline{k}, \gamma) = \begin{bmatrix} C_\gamma & -S_\gamma & 0 \\ S_\gamma & C_\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$Rot(\underline{u}, \theta) \Rightarrow$ formula generale sul testo

— o —

$Rot(\underline{u}, \theta)$ $\quad$ (2.35)

$$Rot(\underline{u}, -\theta) = R(\underline{u}, 2\pi - \theta) = R(-\underline{u}, \theta)$$



VERSORE = VETTORE $\|\cdot\| = 1$

$$\|\underline{u}\| = 1$$

$$\underline{u}^T \underline{u} = 1$$

$$\underline{u}' = -\underline{u}$$

$$Rot(\underline{u}, \theta) \cdot Rot(\underline{u}', \phi) \cdot Rot(\underline{u}'', \psi)$$

$$R_1 R_2 R_3 \cdots R_n$$

$$R_1 R_2 \neq R_2 R_1$$

IN GENERALE

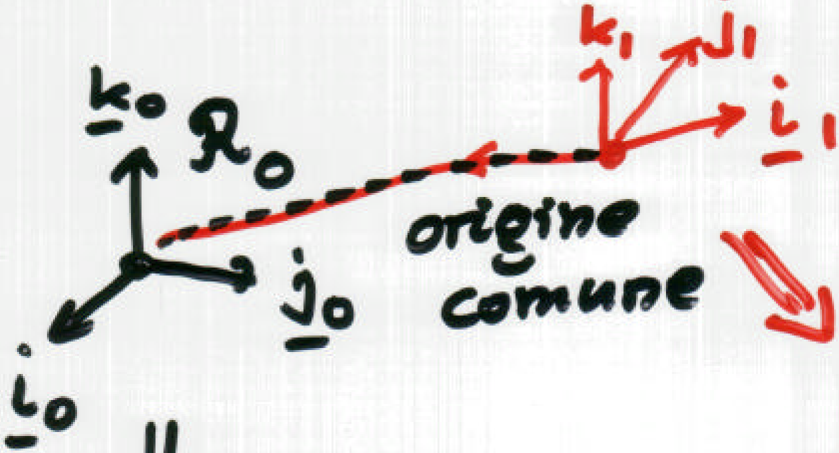
$$Rot(\underline{u}, \theta) \cdot Rot(\underline{u}, \phi) \cdot \dots \cdot Rot(\underline{u}, \psi)$$

$$= Rot(\underline{u}, (\theta + \phi + \dots + \psi))$$

$$Rot(\underline{u}, \phi)\, Rot(\underline{u}, \psi) \cdots Rot(\underline{u}, \theta)$$

SOLO ROT. PIANE

$$R_1 R_2 R_3 R_4 = R_2 R_3 R_4 R_1$$



$$\Rightarrow \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}_{\mathcal{R}_1}$$

$$\Downarrow$$

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}_{\mathcal{R}_0}$$

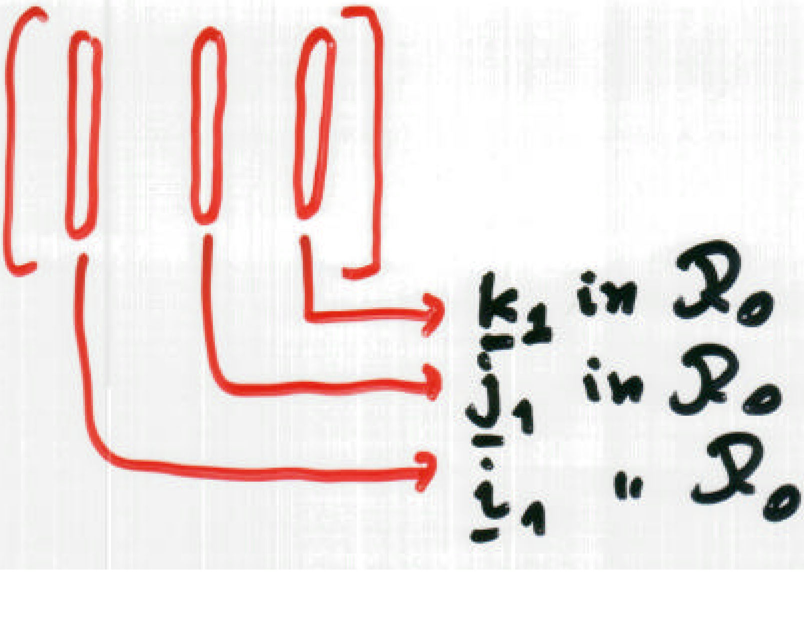
$\mathcal{R}_0$ in $\mathcal{R}_1$

oppure

$\mathcal{R}_1$ in $\mathcal{R}_0$

se $\mathcal{R}_0$ è quello "fisso"

$\mathcal{R}_1$ in $\mathcal{R}_0$ cos'è ?



ESEMPIO

$Rot(\underline{i}_0, 45°)$

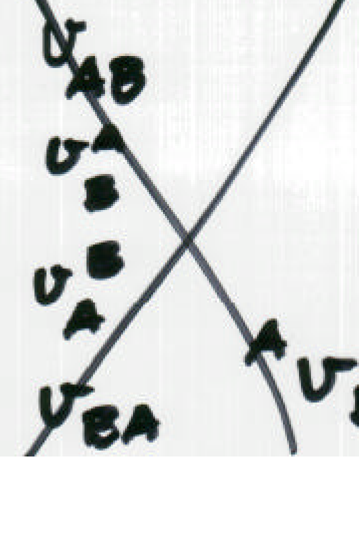
$$\begin{array}{cccc} & \underline{i}_1 & \underline{j}_1 & \underline{k}_1 \\ & \downarrow & \downarrow & \downarrow \\ R = & \multicolumn{3}{c}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{2}/2 & -\sqrt{2}/2 \\ 0 & \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}} \end{array}$$

R rappresenta $\mathcal{R}_1$ in $\mathcal{R}_0$ $\rightarrow$ R^0_1 SI

$^0R_1$ NO

R_{01} NO

R_{10} NO



R_1^0 che rappresenta $\mathcal{R}_1$ in $\mathcal{R}_0$
è anche la trasformazione "fisica"
che porta $\mathcal{R}_0$ a sovrapporsi a $\mathcal{R}_1$

—o—

$$R = I R = R I$$

$$\boxed{\begin{array}{l}\text{PRE-FISSO}\\ \text{POST-MOBILE}\end{array}}$$