ROTO-TRASLAZIONI

R^{i}_{j}

i ← SISTEMA "FISSO" $\mathcal{R}_i$

j ← " "MOBILE" $\mathcal{R}_j$

matrice 3x3   3D

•) TRASFORMA UN VETTORE

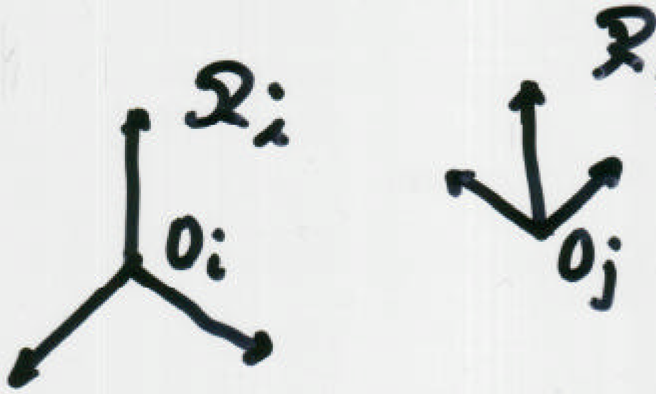
$$[\underline{x}]_{\mathcal{R}_j} \longrightarrow [\underline{x}]_{\mathcal{R}_i} = R^{i}_{j} [\underline{x}]_{\mathcal{R}_j}$$

•) R^{i}_{j} contiene, per colonne

$$\begin{bmatrix} | & | & | \end{bmatrix}$$ i versori di $\mathcal{R}_j$

rappresentati in $\mathcal{R}_i$

•) R^{i}_{j} descrive la rotazione che porta $\mathcal{R}_i$ a sovrapporsi a $\mathcal{R}_j$



$O_i \equiv O_j$

$$R^i_j \longrightarrow \left[R^i_j\right]^{-1} \equiv \left[R^i_j\right]^T = R^j_i$$

$$Rot(\underline{i}, 45) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{2}/2 & -\sqrt{2}/2 \\ 0 & \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix} = R_1$$

$$Rot(\underline{i}, -45°) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{2}/2 & \sqrt{2}/2 \\ 0 & -\sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix} = R_1^T$$

una è l'inverso
dell'altra

COME COMPONGO n ROTAZIONI

1) individuo la rotazione
2) individuo se la rotazione avviene rispetto agli assi "fissi" oppure "mobili:"

3) PRE MOLTIPLICO SE "FISSO"
POST MOLTIPLICO SE "MOBILI"

ESEMPIO

ANGOLI DI EULERO

1) $Rot(\underline{k}, \phi)$ assi mobili
2) $Rot(\underline{i}, \theta)$ " "
3) $Rot(\underline{k}, \psi)$ " "

$$Rot(\underline{k}, \phi) = R_1 = \begin{bmatrix} C\phi & -S\phi & 0 \\ S\phi & C\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Rot(\underline{i}, \theta) = R_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C_\theta & -S_\theta \\ 0 & S_\theta & C_\theta \end{bmatrix}$$

$$Rot(\underline{k}, \psi) = R_3 = \begin{bmatrix} C_\psi & -S_\psi & 0 \\ S_\psi & C_\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(I) $R_1 R_2 R_3$

$$Rot_{EULERO} = R_1 R_2 R_3 \quad (2.73)$$

$R_a \; R_b \; R_c$

1 2 3

3 1 2

2 1 3

3 2 1

R_c

$R_b R_c$

$R_a(R_b R_c)$

(R_b)

$(R_b R_c)$

$R_a(R_b R_c)$

R_b

$R_a R_b$

$(R_a R_b) R_c$

$R'_c R''_c R''_c$

$$R_a = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$R_c = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$R_b = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \qquad R_a R_b R_c$$

ROTO + TRASLAZIONE

$\Downarrow$

VETTORI OMOGENEI
(HOMOGENEOUS VECTORS)

$$\tilde{x} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

STRUTTURALE

$$x \to \tilde{x} = \begin{bmatrix} x \\ \cdots \\ 1 \end{bmatrix}$$

$$\tilde{x} + \tilde{y} = \ ? = \begin{bmatrix} x+y \\ \cdots\cdots \\ 2 \end{bmatrix} \ !!! \quad \text{NO} \quad = \begin{bmatrix} x+y \\ \cdots \\ 1 \end{bmatrix}$$

SI

ROTO-TRASLAZIONI = MATRICI 4×4

$$T = \left[\begin{array}{c|c} R_{3\times3} & t_{3\times1} \\ \hline 0 \ 0 \ 0 & 1 \end{array}\right]$$

COME SI ROTOTRASLA UN VETTORE $\underline{x}$?

1) $\underline{x} \to \underline{\tilde{x}}$

2) $\underline{\tilde{x}}' = T\underline{\tilde{x}}$

3) $\underline{\tilde{x}}' \to \underline{x}'$

$$T = T_1 T_2 T_3 T_4 T_5 \cdots$$

PRE - FISSO
POST - MOBILE

$$T \overset{?}{=} Rot(\underline{i}, \alpha) \qquad T = \begin{bmatrix} R_{\underline{i},\alpha} & \underline{0} \\ \underline{0}^T & 1 \end{bmatrix}$$

$$T \overset{?}{=} Rot(\underline{j}, \beta) = \quad T = \begin{bmatrix} R_{\underline{j},\beta} & \underline{0} \\ \underline{0}^T & 1 \end{bmatrix}$$

Rotazione semplice R

$$T = \begin{bmatrix} R & \underline{0} \\ \underline{0}^T & 1 \end{bmatrix}$$

Traslazione semplice $\underline{t}$

$$T = \begin{bmatrix} I & \underline{t} \end{bmatrix}$$

$$T_a = \begin{bmatrix} R & \underline{0} \\ \underline{0}^T & 1 \end{bmatrix} \qquad T_b = \begin{bmatrix} I & \underline{t} \\ \underline{0}^T & 1 \end{bmatrix}$$

$T_a T_b$

$T_b T_a$

I) $T_a T_b$
- Rot + trasl mobile
- trasl + Rot fisso

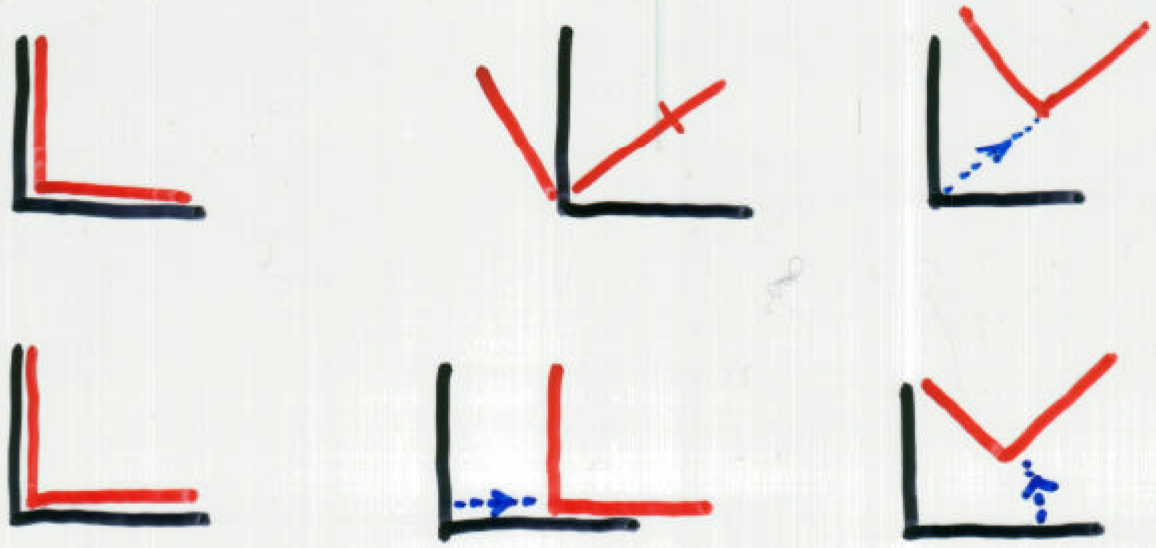
$$\begin{bmatrix} R & \underline{0} \\ \underline{0}^T & 1 \end{bmatrix} \begin{bmatrix} I & \underline{t} \\ \underline{0}^T & 1 \end{bmatrix} = \begin{bmatrix} R & Rt \\ 0^T & 1 \end{bmatrix}$$

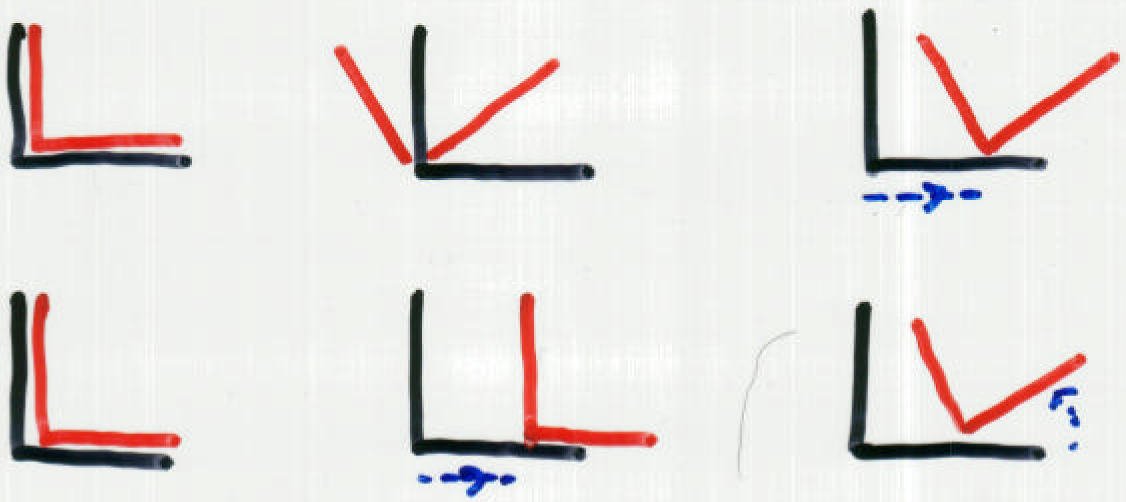
$$\underline{0}\,\underline{0}^T = \begin{bmatrix} \\ \\ \end{bmatrix} \begin{bmatrix} & & \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$a^T b$

$a b^T$

II) $T_b T_a$ $\quad \begin{bmatrix} I & \underline{t} \\ \underline{0}^T & 1 \end{bmatrix} \begin{bmatrix} R & \underline{0} \\ \underline{0}^T & 1 \end{bmatrix} = \begin{bmatrix} R & t \\ 0^T & 1 \end{bmatrix}$
- Rot + trasl fisso
- trasl + Rot mobile





R matrice di rotazione $\underbrace{3 \times 3}_{\substack{9 \\ \text{elementi}}}$

9 elementi, ma solo 3 parametri sono essenziali

data R come estrarre i 3 elementi ?

esistono altri modi di rappresentare l'assetto ?

— o —

dare i 3 elementi direttamente
↓
angoli

1) R 9elem
2) ANGOLI < DI EULERO / RPY
   3 angoli
3) ASSE - ANGOLO $\underline{u}$ θ 1 per
   $\|\underline{u}\| = 1$ 2 per

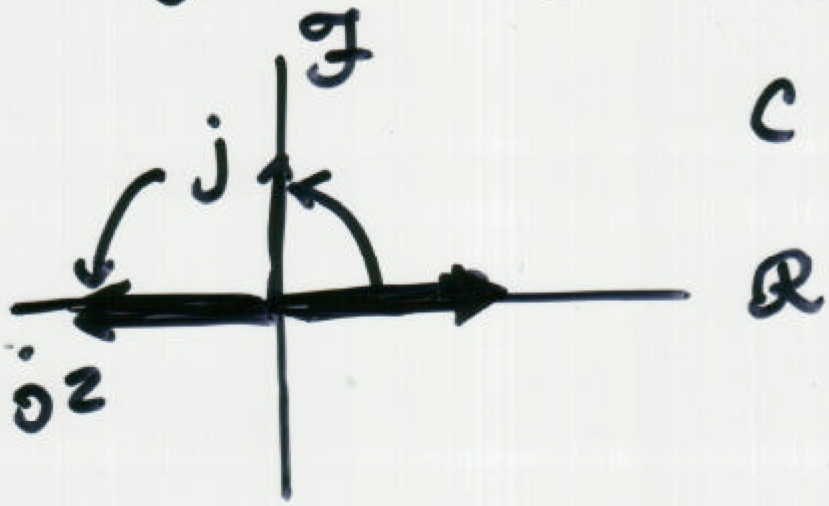
4) QUATERNIONI (UNITARI)

5) VETTORI DI ROTAZIONE DI RODRIGUES

6) MATRICI DI DIRAC 2×2

QUATERNIONI

$a + jb \qquad j^2 = -1$



$h = h_0 + i\,h_1 + j\,h_2 + k\,h_3$

$i^2 = j^2 = k^2 = -1$

$ij = -ji = k \quad jk = -kj = i \quad ki = -ik = j$

$ijk = 1$

$$h = \underbrace{h_0}_{\text{PARTE REALE}} + \underbrace{i h_1 + j h_2 + k h_3}_{\text{PARTE VETTORIALE}}$$

$h = (h_0, 0, 0, 0)$ reali
$h = (h_0, h_1, 0, 0)$ complessi
$h = (0, h_1, h_2, h_3)$ vettori

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norma di quaternione

$$\|h\| = \sqrt{h_0^2 + h_1^2 + h_2^2 + h_3^2}$$

unitario $\|h\| = 1$

$$\underline{h} = \sin\frac{\theta}{2} + i\, u_1 \cos\frac{\theta}{2} + j\, u_2 \cos\frac{\theta}{2} + k\, u_3 \cos\frac{\theta}{2}$$

$\|h\| = 1$

n rotazioni $R_1 \cdots R_n$

$$\begin{array}{ccc} R_1 & \cdots & R_n \\ \downarrow & & \downarrow \\ h_1 & & h_n \end{array}$$

$R_1 \cdots R_n = R \longrightarrow h$

$$h = h_1 h_2 \cdots h_n$$

— o —