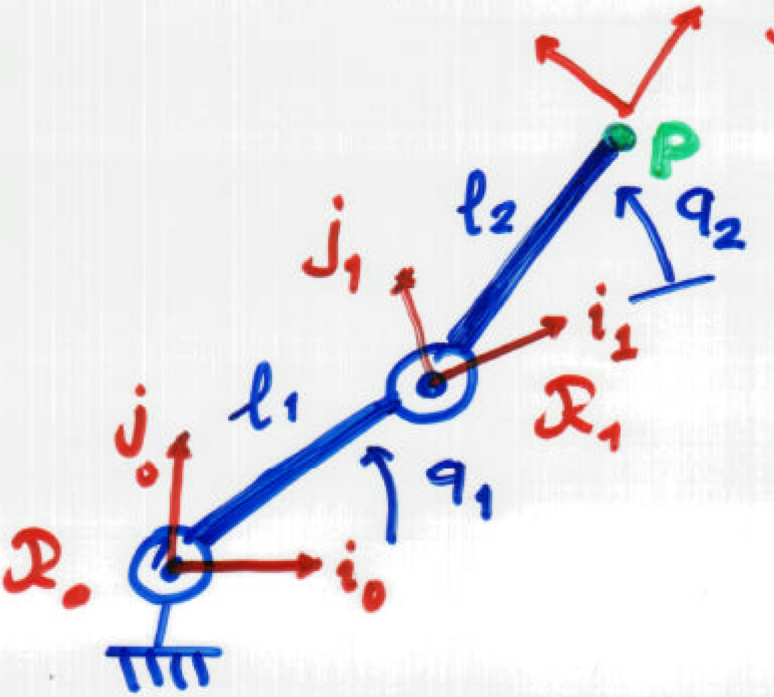
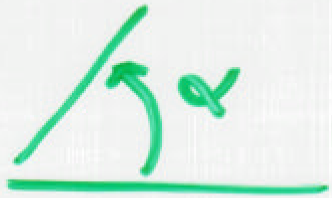




$c_i = \cos q_i$

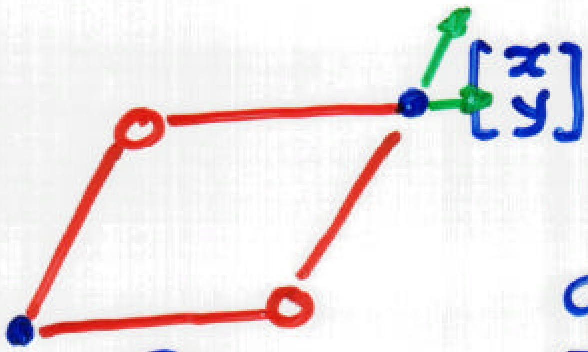
$c_{ij} = \cos(q_i + q_j)$

$s_i = \sin q_i$

$s_{ij} = \sin(q_i + q_j)$

$$\begin{cases} x_p = \dots = l_1 c_1 + l_2 c_{12} \\ y_p = \dots = l_1 s_1 + l_2 s_{12} \\ \alpha = \dots = q_1 + q_2 \end{cases}$$

$\underline{q}$ NO



$$\underline{q}(t) \rightarrow p(t) = \begin{bmatrix} x \\ y \\ \alpha \end{bmatrix}$$

AN. INV. ?

$$\begin{bmatrix} x \\ \alpha \end{bmatrix} \rightarrow \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \qquad \begin{bmatrix} x \\ y \end{bmatrix} \rightarrow \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

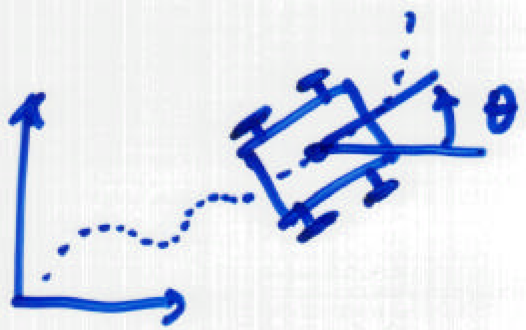
$$\begin{bmatrix} y \\ \alpha \end{bmatrix} \rightarrow \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

CIN. DIR. POS. FACILE

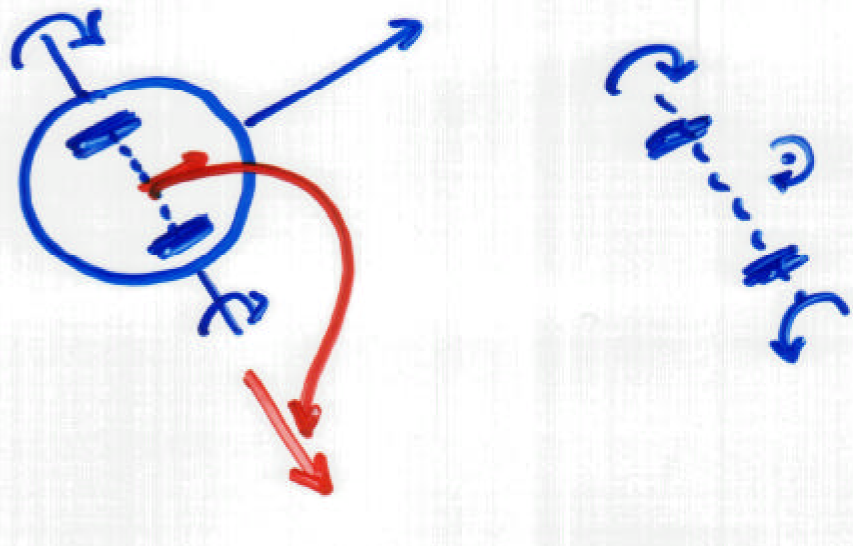
CIN. INV. POS. DIFFICILE, MA NECESSARIO

$$q_i(t) = f_i(\underline{p})$$

UTILE/NECESSARIO X IL CONTROLLO



$$\begin{bmatrix} x \\ y \\ \theta \end{bmatrix} = \underline{p} \qquad q_i\,?$$



— ODOMETRIA —

MISURA NUMERO GIRI MOTORE

CIN. DIR. VEL.
CIN. INV. VEL.

$$\underline{p} = \underline{f}(\underline{q}) \rightarrow \dot{\underline{p}} = \frac{d}{dt}\left[\underline{f}(\underline{q}(t))\right]$$

$$\dot{p}_1 = \frac{d}{dt}\left[f_1(\underline{q}(t))\right]$$

$$\dot{p}_2 = \frac{d}{dt}\left[f_2(\underline{q}(t))\right]$$

$$\vdots$$

$$\dot{\underline{p}} = \underbrace{\begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}}_{J} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \vdots \end{bmatrix}$$

jacobiano della trasformazione

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial q_1} & \frac{\partial f_1}{\partial q_2} & \cdots & \frac{\partial f_1}{\partial q_n} \\ & & \vdots & \\ \frac{\partial f_m}{\partial q_1} & & \cdots & \frac{\partial f_m}{\partial q_n} \end{bmatrix}$$

$$\dot{p} = J(\underline{q})\dot{q}$$

$$x = lc_1 + lc_{12} \qquad l_1 = l_2 = l$$

$$y = ls_1 + ls_{12}$$

$$\alpha = q_1 + q_2$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\alpha} \end{bmatrix} = \begin{bmatrix} -ls_1\dot{q}_1 - ls_{12}(\dot{q}_1 + \dot{q}_2) \\ lc_1\dot{q}_1 + lc_{12}(\dot{q}_1 + \dot{q}_2) \\ \dot{q}_1 + \dot{q}_2 \end{bmatrix}$$

$$\begin{bmatrix}\dot x\\ \dot y\\ \dot\alpha\end{bmatrix} = \underbrace{\begin{bmatrix} -\ell s_1 - \ell s_{12} & -\ell s_{12}\\ \ell c_1 + \ell c_{12} & \ell c_{12}\\ 1 & 1\end{bmatrix}}_{J(q_1,\,q_2)} \begin{bmatrix}\dot q_1\\ \dot q_2\end{bmatrix}$$

$$\frac{\partial f_1}{\partial q_1} = -\ell s_1 - \ell s_{12} \qquad \frac{\partial f_1}{\partial q_2} = -\ell s_{12}$$

$$\frac{\partial f_2}{\partial q_1} = \ell c_1 + \ell c_{12} \qquad \frac{\partial f_2}{\partial q_1} = \ell c_{12}$$

$$\frac{\partial f_3}{\partial q_1} = 1 \qquad \frac{\partial f_3}{\partial q_2} = 1$$

CIN. INV. VEL. ?

$$\dot p = J\dot q \rightarrow \dot q = J^{-1}\dot p \quad !!$$

SOLO SE QUADRATA !

$m = n$

$6 = gd\ell = gdm = 6$

$$\underset{n \times n}{J^{-1}}(\underline{q}) = \frac{1}{\det J(q)} \cdot \left[\ ?\ \right]$$

$$\dot{q} = J^{-1}\dot{p}$$

$$\det J(\bar{q}) = 0$$

$\bar{q}$ = CONFIGURAZIONE SINGOLARE

$J(q)$ cade di rango!

rank = 5

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -\ell s_1 - \ell s_{12} & -\ell s_{12} \\ \ell c_1 + \ell c_{12} & \ell c_{12} \end{bmatrix}$$

$$J(q)_{2 \times 2}$$

$\ell = 1$

$$\hookrightarrow J = \begin{bmatrix} -s_1 - s_{12} & -s_{12} \\ c_1 + c_{12} & c_{12} \end{bmatrix}$$

$$\det(J) = -c_{12}(s_1 + s_{12}) + s_{12}(c_1 + c_{12})$$

$$\det(J) = 0 = \underbrace{s_{12}c_{12} \cancel{-} s_{12}c_{12}}_{=0}$$

$$-s_1 c_{12} + c_1 s_{12}$$

$$s_{12}c_1 - c_{12}s_1 = \sin(\underbrace{q_1 + q_2}_{\alpha} \underbrace{- q_1}_{-\beta})$$

$$\det(J) = \sin(q_2)$$

è singolare quando $q_2 = 0°$

$q_2 = 180°$

$\vdots$