- CINEMATICA
- DINAMICA
- (STATICA)
- CONTROLLO - PIANIFICAZIONE DEL MOVIMENTO
  - TRAIETTORIE

CARTESIANO : $\underline{p}(t)$, $\dot{\underline{p}}(t)$, $\ddot{\underline{p}}(t)$

MONDO DEI COMANDI: $\underline{q}(t)$, $\dot{\underline{q}}(t)$, $\ddot{\underline{q}}(t)$

$$\frac{d}{dt}\left[\dot{\underline{p}}(t)\right] = \frac{d}{dt}\left[J(q(t))\,\dot{\underline{q}}(t)\right]$$

$$= J(q(t))\,\ddot{q}(t) + \frac{d}{dt}\left[J(\underline{q(t)})\right]\dot{q}(t)$$

$$= J(q(t))\,\ddot{q}(t) + \frac{d}{dt}\begin{bmatrix} J_{11}(q) & \\ & \end{bmatrix}\begin{bmatrix}\dot{q}_1\\ \vdots \\ \dot{q}_n\end{bmatrix}$$

$$\frac{d}{dt}\left[J(q(t))\right]\dot{q}(t) =$$

$$\begin{bmatrix} \frac{d}{dt}J_{11}(\underline{q}(t)) & \cdots & \\ \vdots & & \\ & & \frac{d}{dt}J_{mn}(\cdot) \end{bmatrix} \begin{bmatrix} \dot{q}_1(t) \\ \cdots \\ \dot{q}_n(t) \end{bmatrix}$$

$$\frac{d}{dt}J_{11}(\underline{q}(t)) = \frac{\partial}{\partial q_1}J_{11}(\underline{q})\frac{dq_1}{dt} + \frac{\partial}{\partial q_2}J_{11}(\underline{q})\frac{dq_2}{dt}$$

$$+ \cdots$$

$H_1(\underline{q})\,\dot{\underline{q}}^2 +$ termini misti $\dot{q}_i \dot{q}_j$

$$\dot{x} = -\ell s_1 \dot{q}_1 - \ell s_{12}(\dot{q}_1 + \dot{q}_2)$$

$$= -\ell(s_1 + s_{12})\dot{q}_1 - \ell s_{12}\dot{q}_2$$

$$\ddot{x} = \frac{d}{dt}(-\ell s_1 \dot{q}_1) + \frac{d}{dt}(-\ell s_{12}\dot{q}_1) + \frac{d}{dt}(-\ell s_{12}\dot{q}_2)$$

$$= -\ell c_1 \dot{q}_1^2 - \ell s_1 \ddot{q}_1$$

$$- \ell c_{12}(\dot{q}_1 + \dot{q}_2)\dot{q}_1 - \ell s_{12}\ddot{q}_1$$

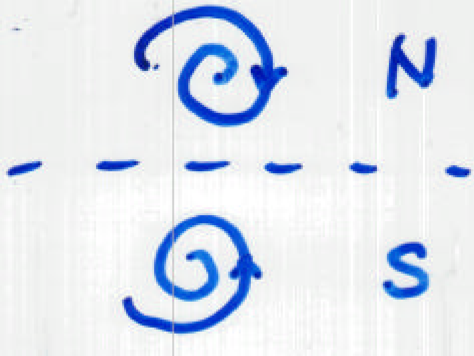
$$- \ell c_{12}\dot{q}_1^2 - \ell c_{12}\dot{q}_1\dot{q}_2 - \ell s_{12}\ddot{q}_1$$

TERMINE B   TERMINE C   TERMINE A

A : EFFETTI "INERZIALI"

B : " "CENTRIFUGHI"

C : " "CORIOLIS"



$\underline{x} = \begin{bmatrix} \vdots \end{bmatrix}$

$\mathcal{R}_0$ $\mathcal{R}_1(t)$

↑ FISSO ↑ MOBILE

$\underline{x}_0 = \underset{R(t)}{\underset{\downarrow}{R^0_1}} \, \underline{x}_1(t) + \underset{\underline{t}(t)}{\underset{\downarrow}{\boxed{t^0_1}}}$

$\dot{\underline{x}}_0 = ?$

$= R\,\dot{\underline{x}}_1(t) + \dot{R}\,\underline{x}_1(t) + \dot{\underline{t}}(t)$

$\ddot{\underline{x}}_0$

$$\dot{R}(t) = ?$$

$$R^{-1} = R^T$$

$RR^{-1} = R^{-1}R = I$ sempre $\forall t$

$$R(t)R^T(t) = R^T(t)R(t) = I$$

$\frac{d}{dt}(I)$

$$\frac{d}{dt}(RR^T) = \underline{0}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\underbrace{\dot{R}R^T}_{S} + \underbrace{R\dot{R}^T}_{S^T} = \underline{0}$$

$$S + S^T = 0$$

$$(A \cdot B \cdot C)^T = C^T \cdot B^T \cdot A^T$$

$M = M^T$
SIMMETRICA

$$S + S^T = 0$$

$$S = -S^T$$

ANTISIMMETRICA

$$S = \begin{bmatrix} 0 & x & \square \\ -x & 0 & \triangle \\ -\square & -\triangle & 0 \end{bmatrix} = \begin{bmatrix} 0 & -u_3 & u_2 \\ u_3 & 0 & -u_1 \\ -u_2 & u_1 & 0 \end{bmatrix}$$

$$\underline{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \rightsquigarrow S(\underline{u})$$

1) $S(\underline{u})\,\underline{v} = \underline{u} \times \underline{v}$

2) $R = I + \dfrac{\sin\theta}{\theta} S(\underline{v}) + \dfrac{1-\cos\theta}{\theta^2} S^2(\underline{v})$

(2.125)

$$= e^{S(\underline{v})} = e^{S(\theta\,\underline{u})}$$

$$\underbrace{\|\underline{v}\| = \theta \qquad \frac{v}{\|v\|} = u = \text{asse di rotazione}}_{\text{asse - angolo}}$$

$\dot{R} = SR$

$S(?)$ $\quad$ $R(\theta(t), \underline{u})$

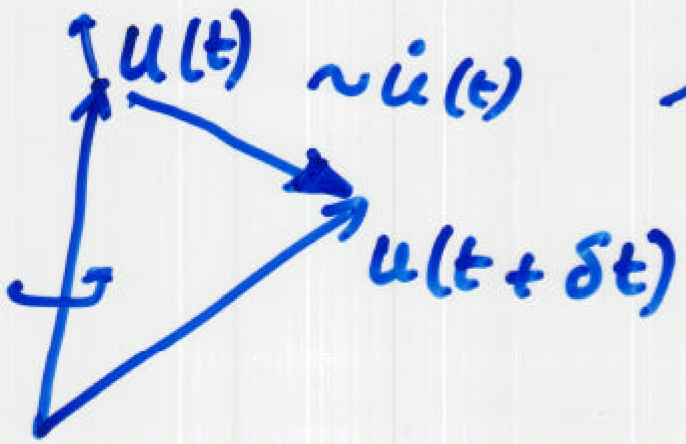
$S(\underline{\omega})$ $\quad$ $\underline{\omega}$ = vel. ang. istantanea

$\dot{R}(t) = S(\underline{\omega})R(t)$ $\quad$ $R \to \underline{u}, \theta$

$$\underline{\omega}(t) = \dot{\theta}(t)\,\underline{u}(t) + \sin\theta(t)\,\dot{\underline{u}}(t) \cdot + (1-\cos\theta(t)) \cdot S(\underline{u}(t))\,\dot{\underline{u}}(t) \qquad (2.127)$$

se $\dot{\underline{u}}(t) = 0$

$$\underline{\omega}(t) = \dot{\theta}\,\underline{u}(t)$$



se $\delta t = \dots$

$$\underline{\dot{u}}(t) \simeq \frac{u(t+\delta t) - u(t)}{\delta t}$$

CASO A : $\underline{u}$ costante o quasi

$$\dot{R} = S(\omega)R \qquad \text{ma } \omega \simeq \dot{\theta} u$$

$$= S(\dot{\theta} u) R =$$

$$= \dot{\theta} S(u) R$$

$$= \dot{\theta} \begin{bmatrix} \underline{u} \times \underline{c}_1 & \underline{u} \times c_2 & \underline{u} \times c_3 \end{bmatrix}$$

$$\dot{\underline{x}}_0 = R\dot{\underline{x}}_1 + \dot{R}\underline{x}_1 + \dot{\underline{t}}$$

$$= R\dot{\underline{x}}_1 + S(\omega)R\underline{x}_1 + \dot{\underline{t}}$$

$\underline{\omega} \times \underline{r}$

$$\ddot{\underline{x}}_0 = \dot{R}\dot{x}_1 + R\ddot{x}_1 + \ddot{t}$$

$$\boxed{\begin{array}{l}\dot{S}(\omega)Rx_1 + \\ S(\omega)\dot{R}x_1 + \\ S(\omega)R\dot{x}_1\end{array}}$$

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$S(\omega)Rx_1 \rightarrow \underline{\omega} \times (R\underline{x}_1)$

$$\frac{d}{dt}\left[\omega \times (Rx_1)\right] = \boxed{\begin{array}{l}\dot{\omega} \times (Rx_1) + \\ \omega \times \frac{d}{dt}(Rx_1)\end{array}}$$

$\dot{S}(\omega) = S(\dot{\omega})$