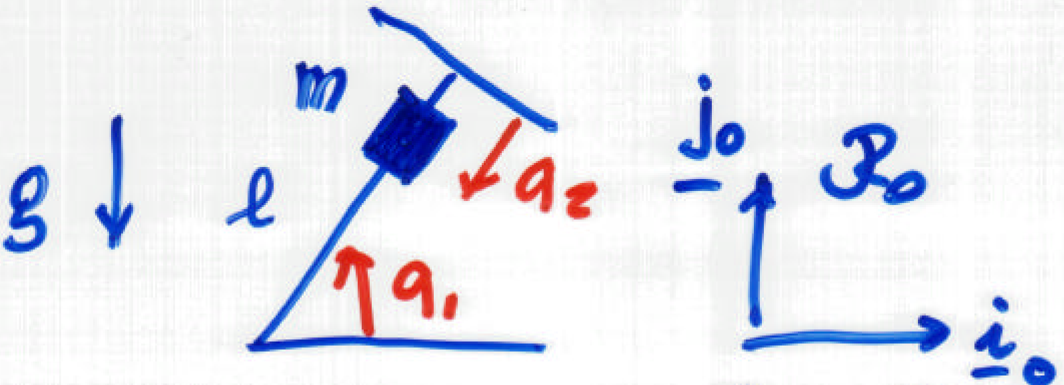



$$\underline{x} = \begin{bmatrix} (\ell - q_2)\cos q_1 \\ (\ell - q_2)\sin q_1 \end{bmatrix} \qquad \downarrow g = \begin{bmatrix} 0 \\ -10 \\ 0 \end{bmatrix}$$

$$\underline{\dot{x}} = \begin{bmatrix} -\ell \sin q_1 \dot{q}_1 - \cos q_1 \dot{q}_2 + \sin q_1 q_2 \dot{q}_1 \\ \ell \cos q_1 \dot{q}_1 - \sin q_1 \dot{q}_2 - \cos q_1 \dot{q}_1 q_2 \end{bmatrix}$$

$$\Downarrow$$

$$\underline{\dot{p}} = J \underline{\dot{q}} = \cancel{\begin{bmatrix} -(\ell - q_2) s_1 & \cdots \\ & \end{bmatrix}}$$

$$\underbrace{\begin{bmatrix} -(\ell - q_2) s_1 & -c_1 \\ (\ell - q_2) c_1 & -s_1 \end{bmatrix}}_{J(q_1, q_2)} \underbrace{\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}}_{\underline{\dot{q}}}$$

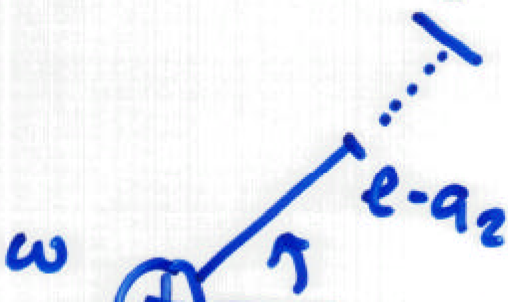
$$C = \frac{1}{2}\|v_c\|^2 \cdot m + \underbrace{\frac{1}{2}\omega^T \Gamma \omega}_{=0}$$

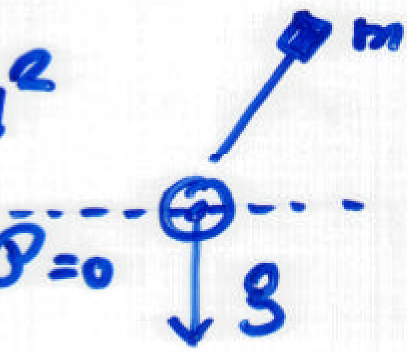
$$\|v_c\|^2 = \dot{q}^T J^T J \dot{q} = \dot{p}^T \dot{p}$$

$$J^T J = \begin{bmatrix} (l-q_2)^2 & 0 \\ 0 & 1 \end{bmatrix}$$

$$[\dot{q}_1, \dot{q}_2]\begin{bmatrix} (l-q_2)^2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

$$(l-q_2)^2 \dot{q}_1^2 + \dot{q}_2^2 = \|v_c\|^2$$



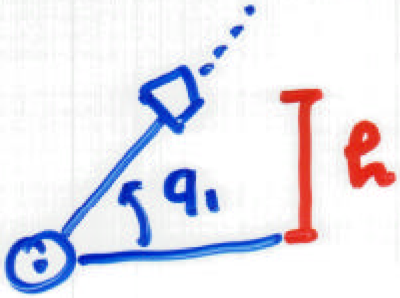




$$C = \frac{1}{2} m \left[(l-q_2)^2 \dot{q}_1^2 + \dot{q}_2^2\right]$$

$$P = -m g^T r_{0,c} = -m\,[0 \;\; \underset{G}{-10} \;\; 0]\begin{bmatrix} (l-q_2)c_1 \\ (l-q_2)s_1 \\ 0 \end{bmatrix}$$

$$m \cdot G (l-q_2) s_1$$

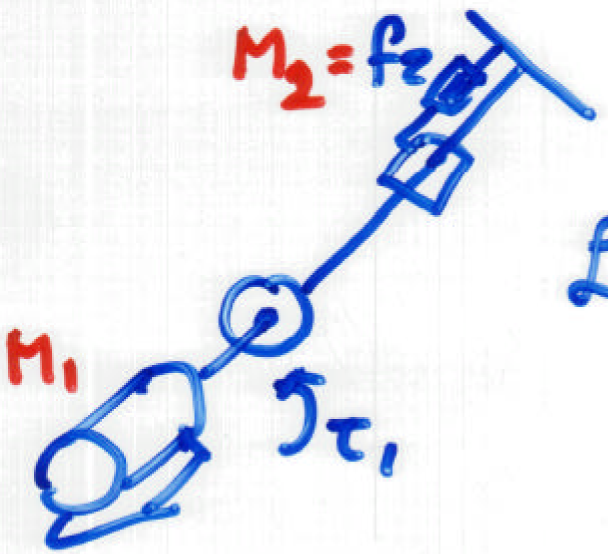
$$\mathcal{P} = 10 \cdot m \underbrace{(\ell - q_2) s_1}_{h}$$




$$\mathcal{D} = \frac{1}{2}\beta \dot{q}_2^2 \qquad \mathcal{F}_1 = ? = \tau_1$$

$$\mathcal{F}_2 = ? = f_2$$

se ci sono entrambi i motori $M_i$
altrimenti $\mathcal{F}_i$ cambiano




$$\mathcal{L} = \mathcal{C} - \mathcal{P} = \dots$$

$$\frac{d}{dt}\left[\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right] - \frac{\partial \mathcal{L}}{\partial q_i} + \frac{\partial D}{\partial \dot{q}_i} = \mathcal{F}_i$$

---

$i=1 \quad \dot{q}_1 ; q_1$

$$\overset{1}{\frac{\partial C}{\partial \dot{q}_1}} \quad \overset{2}{\frac{\partial P}{\partial \dot{q}_1}} \quad \overset{3}{\frac{\partial C}{\partial q_1}} \quad \overset{4}{\frac{\partial P}{\partial q_1}} \quad \overset{5}{\frac{\partial D}{\partial \dot{q}_1}}$$

1) $\frac{1}{2} m \cdot 2 (\ell - q_2)^2 \dot{q}_1$     2) $= 0$

3) $0$     4) $mG(\ell - q_2) c_1$     5) $0$

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$$\frac{d}{dt}\left[m(\ell - q_2)^2 \dot{q}_1\right] + mG(\ell - q_2) c_1 = \tau_1$$

$$m(\ell - q_2)^2 \ddot{q}_1 - 2m(\ell - q_2)\dot{q}_1 \dot{q}_2 + mG(\ell - q_2) c_1 = \tau_1$$

$i=2 \quad \dot{q}_2 ; q_2$

$$\overset{1}{\frac{\partial \mathcal{C}}{\partial \dot{q}_2}} \quad \overset{2}{\frac{\partial \mathcal{P}}{\partial \dot{q}_2}} \quad \overset{3}{\frac{\partial \mathcal{C}}{\partial q_2}} \quad \overset{4}{\frac{\partial \mathcal{P}}{\partial q_2}} \quad \overset{5}{\frac{\partial \mathcal{D}}{\partial \dot{q}_2}}$$

1) $\frac{1}{2} m 2 \dot{q}_2$ 2) $0$

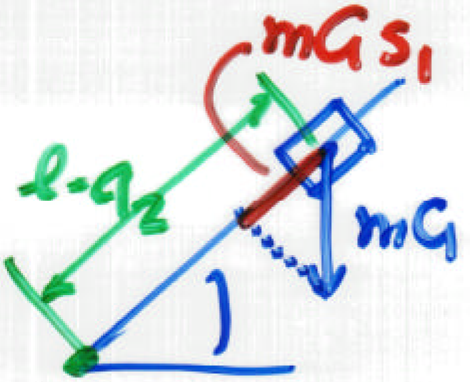
3) $-\frac{1}{2} m \dot{q}_1^2 2(\ell - q_2)$ 4) $-mG s_1$

5) $\beta \dot{q}_2$

$$\frac{d}{dt}\left[ m\dot{q}_2 \right] + m(\ell - q_2)\dot{q}_1^2 - mG s_1 + \beta \dot{q}_2 = f_2$$

$$\boxed{\begin{aligned} & m\ddot{q}_2 + m(\ell - q_2)\dot{q}_1^2 - mG s_1 + \beta \dot{q}_2 = f_2 \\ & m(\ell - q_2)^2 \ddot{q}_1 - 2m(\ell - q_2)\dot{q}_1 \dot{q}_2 + mG(\ell - q_2) c_1 = \tau_1 \end{aligned}}$$

$$m\ddot{q}_2 + \underbrace{m(\ell-q_2)\dot{q}_1^2}_{mr\dot{\theta}^2} + \beta\dot{q}_2 = f_2 + mGs_1$$

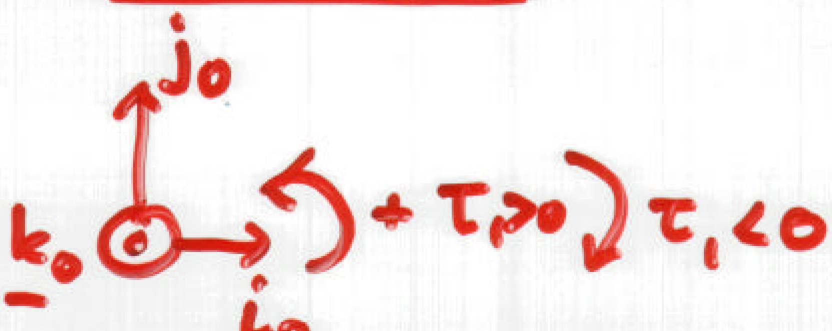


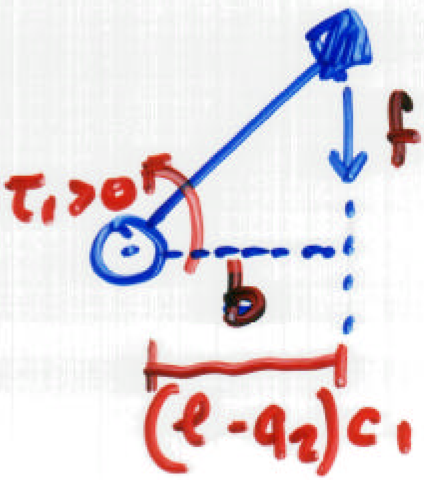


$$\underbrace{m(\ell-q_2)^2}_{m\ell^2\dot{\omega}}\ddot{q}_1 - 2m(\ell-q_2)\dot{q}_1\dot{q}_2$$

$$= \tau_1 - \underline{\overbrace{mG}^{f\cdot b}(\ell-q_2)c_1}$$

$I\dot{\omega}$






$\curvearrowright fb < 0$

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PASSAGGIO IN VARIABILI DI STATO

È UTILE? < SIMULAZIONE / CONTROLLO

---

$$q_i \rightarrow \begin{bmatrix} q_1 \\ \vdots \\ q_n \end{bmatrix} \qquad \dot{q}_i = \begin{bmatrix} \dot{q}_1 \\ \vdots \\ \dot{q}_n \end{bmatrix}$$

$\underline{x}$ STATO

$\underline{\xi}$ $\underline{z}$

$$\underline{x} = \begin{bmatrix} \underline{q} \\ \underline{\dot{q}} \end{bmatrix}_{2n} \qquad \underline{z} = \begin{bmatrix} q_1 \\ \dot{q}_1 \\ q_2 \\ \dot{q}_2 \\ \vdots \end{bmatrix}$$

$q_1 = x_1 \quad q_2 = x_2$

$\dot{q}_1 = x_3 \quad \dot{q}_2 = x_4$

---

$$\begin{cases} \underline{\dot{x}}(t) = \underline{f}(\underline{x}(t), \underline{u}(t)) \\ \underline{y}(t) = \underline{g}(\underline{x}(t), \underline{u}(t)) \end{cases}$$

$\underline{x}(t)$ STATO

$\underline{u}(t)$ INGRESSO • COMANDO

$\underline{y}(t)$ USCITA • MISURA

$\dot{x}_1 = \dot{q}_1 = x_3$

$\dot{x}_2 = \dot{q}_2 = x_4$

$\dot{x}_3 = \ddot{q}_1 = \ldots\ldots$

$\dot{x}_4 = \ddot{q}_2 = \ldots\ldots \rightarrow$ ESEMPIO

$m\ddot{q}_2 + m(\ell - q_2)\dot{q}_1^2 + \beta\dot{q}_2 = f_2 + mGs_1$

$$\ddot{q}_2 = \frac{1}{m}\left[f_2 + mGs_1 - m(\ell - q_2)\dot{q}_1^2 - \beta\dot{q}_2\right]$$

$$= \frac{f_2}{m} + G\sin(x_1) - (\ell - x_2)x_3^2 - \frac{\beta}{m}x_4$$

$$f_2 = u_2$$

$$\dot{x}_4 = \frac{1}{m}u_2 + G\sin(x_1) - \ell x_3^2 + x_2 x_3^2 - \frac{\beta}{m}x_4$$

NONLINEARE !!

CONTROLLO

- → LINEARIZZARE
- → USARE TEORIA SIST/CONTR. NONLINEARE ×
- → LINEARIZZARE MEDIANTE CONTROLLO NONLINEARE ××

$$\sin x = x + ? \ldots$$

$$\cos x = 1 + \ldots ?$$

# COME RISULTA LA GENERICA EQUAZIONE DINAMICA DI UN ROBOT

i-esima eq. di Lagrange (5.55)

$$\underbrace{H_{ii}(\underline{q})\ddot{q}_i}_{1} + \underbrace{\sum_{j\neq i} H_{ij}(\underline{q})\ddot{q}_j}_{2} + \underbrace{\sum_{j} h_{ijj}(\underline{q})\dot{q}_j^2}_{3}$$

$$+ \underbrace{\sum_{j}\sum_{k\neq j} h_{ijk}(\underline{q})\dot{q}_k \dot{q}_j}_{4} + \underbrace{g_i(\underline{q})}_{5} = \tau_i$$

COMANDO

1) INERZIA EQUIVALENTE

2) EFFETTO INERZIALE EQUIVALENTE DI VINCOLO TRASMESSO

3) CENTRIFUGO

4) CORIOLIS

5) GRAVITAZIONALE

LE n equazioni si scrivono matric.

$$\overset{1+2}{H(\underline{q})\ddot{\underline{q}}} + \overset{3+4}{C(\underline{q},\dot{\underline{q}})\dot{\underline{q}}} + \overset{5}{g(\underline{q})} = \underline{\tau}$$

$n \times n$