CONTROLLO NONLINEARE ROBOT RIGIDI

INTERAZIONE

$$H(\underline{q})\ddot{q}(t) + C(\underline{q},\dot{\underline{q}})\dot{\underline{q}}(t) +$$

$$+ B(\underline{q})\dot{q}(t) + \underline{g}(\underline{q}) + J^T \underline{F_e}(t)$$

$$= R_m\left(\underline{\tau}_m(t) - \underline{\tau}_p(t)\right)$$

$$R_m = \begin{bmatrix} r_1 & & \phi \\ & \ddots & \\ \phi & & r_n \end{bmatrix}$$ rapp. motoriduzione

$$R_m \underline{\tau}_p = R_m\left(J_m \ddot{q}_m + B_m \dot{q}_m\right)$$

$$\begin{bmatrix} & & \end{bmatrix}\begin{bmatrix} \end{bmatrix} \qquad \ddot{q}_m = R_m \ddot{q}$$

$$\dot{q}_m = R_m \dot{q}$$

$$J_m = \mathrm{diag}\{J_{mi})$$

$$B_m = \mathrm{diag}(\beta_{mi})$$

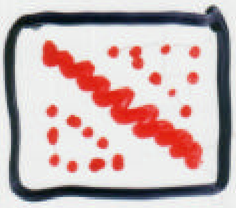
$$R_m \tau_m = R_m K_a v_a - R_m^2 K_\omega \dot{q}$$

QUINDI

$$H\ddot{\underline{q}} + \left[C(\underline{q},\dot{\underline{q}}) + B\right]\dot{\underline{q}} + \underline{g} + J^T \underline{F}_e =$$

$$= R_m K_a v_a - R_m^2 K_\omega \dot{q} - R_m^2 (J_m \ddot{q} + B_m \dot{q})$$

$$\left[H + R_m^2 J_m\right]\ddot{q} + \left[C(q,\dot{q}) + \underbrace{B_e + R_m^2 K_\omega + R_m B_m}_{B_t}\right]\dot{q}$$

 $\ddot{q}$

$$+ \underline{g}(q) + J^T F_e = \underline{u}_c$$

$$\underline{u}_c := R_m K_a v_a$$

MOMENTO

ADIMENS

TENSIONE 2 COPPIA

TENSION

V2C

$$M := H + R_m^2 J_m = M(\underline{q})$$

$$h(\underline{q},\dot{\underline{q}}) := \left[C(q,\dot{q}) + B_t\right]\dot{q}(t) + g(q)$$

$$M(q)\ddot{q} + h(q,\dot{q}) + \underbrace{J^T F_e}_{\text{IPOTESI } = 0} = u_c$$

$$M(\underline{q})\ddot{\underline{q}} + h(q,\dot{q}) = u_c$$

METODO DELLA DINAMICA INVERSA

$$u_c := \hat{M} \cdot (\ddot{q}_r - v_c) + \hat{h}$$

ulteriore comando da progettare

$$M(q)\ddot{q} + h(q,\dot{q}) = \hat{M}(\ddot{q}_r - v_c) + \hat{h}$$

$M(q)$ è invertibile sempre

$$\ddot{q} = M^{-1}(q)\hat{M}(\ddot{q}_r - v_c) - M^{-1}(q)\underbrace{\left[h - \hat{h}\right]}_{\Delta h(q,\dot{q})}$$

$$E(q) = M^{-1}(q)\hat{M} - I$$

$$\boxed{\ddot{q} = (\ddot{q}_r - v_c) + E(q)(\ddot{q}_r - v_c) - M^{-1}(q)\Delta h}$$

[redacted]

$$\ddot{q} = (\ddot{q}_r - v_c) + \eta(q, \dot{q}, \ddot{q}_r, v_c)$$

termine di errore complessivo

VARIABILI DI STATO

$$\underline{x}(t) = \begin{bmatrix} \underline{x}_1 \\ \underline{x}_2 \end{bmatrix} = \begin{bmatrix} \underline{q}(t) \\ \underline{\dot{q}}(t) \end{bmatrix}$$

$$\underline{\dot{x}} = A x + B(\ddot{q}_r - v_c) + B\eta(\cdots)$$

dove

$$\underset{2n\times 2n}{A} = \begin{bmatrix} 0_n & I_n \\ 0_n & 0_n \end{bmatrix}$$

$$B = \begin{bmatrix} 0_n \\ I_n \end{bmatrix}$$

$$e = q_r - q$$
$$\dot{e} = \dot{q}_r - \dot{q}$$
$$\ddot{e} = \ddot{q}_r - \ddot{q}$$

$\Rightarrow$ STATO ERRORE

$$\varepsilon(t) = \begin{bmatrix} \underline{\varepsilon}_1 \\ \underline{\varepsilon}_2 \end{bmatrix} = \begin{bmatrix} \underline{e} \\ \underline{\dot{e}} \end{bmatrix}$$

$$\dot{\varepsilon} = A\varepsilon + Bv_c - B\eta(\cdots)$$

COME DEFINIAMO IL CONTROLLORE

$$\begin{cases} \dot{\xi}(t) = F\,\xi(t) + G\,\varepsilon(t) \\ v_c(t) = H\,\xi(t) + L\,\varepsilon(t) \end{cases}$$

EQUAZIONI

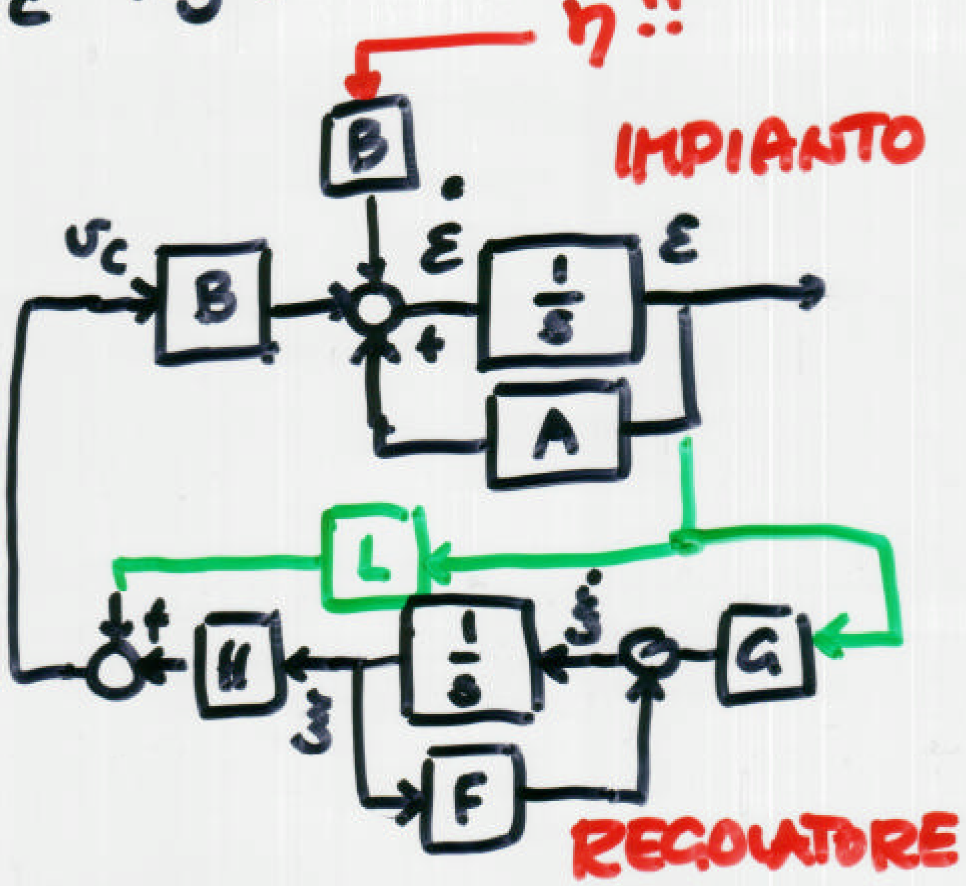
ERRORE

$$\dot{\varepsilon} = A\varepsilon + Bv_c + B\eta$$

!!!

REGOLATORE

$$\begin{cases} \dot{\xi} = F\xi + G\varepsilon \\ v_c = H\xi + L\varepsilon \end{cases}$$



A) PROGETTIAMO u_c

$$u_c := \hat{M}(\ddot{q}_r - v_c) + \hat{h}$$

ANELLO (NONLINEARE) INTERNO

LINEARIZZA GLOBALMENTE
IL SISTEMA

"INNER LOOP"
È INSTABILE

B) CONTROLLO + STABILIZZAZIONE
MEDIANTE CONTROLLORE LINEARE

$$v_c := H\xi + Le$$

"OUTER LOOP"

LINEARIZZAZIONE ESATTA

IPOTESI $\hat{M} = M(q)$

$h(q,\dot{q}) = \hat{h}$

$$\left.\begin{array}{l} M^{-1}\hat{M} = \hat{M}^{-1}M = I \\ \Delta h = \underline{0} \end{array}\right\} \eta(\cdots) = \underline{0}$$

$\underline{u}_c := \ldots$

$\Downarrow$

L'IMPIANTO DIVENTA

$\ddot{q} = \ddot{q}_r - v_c$ DISACCOPPIATO!

OVVERO

$$\underline{\dot{\varepsilon}} = A\underline{\varepsilon} + B\underline{v_c}$$

CON A instabile XCHÈ

TUTTI GLI AUTOV. ~~HANNO~~ TUTTI

~~E~~R. = 0

$\underline{v}_c = ?$

VARIE POSSIBILITA'

- RETE PD

$$\underline{v}_c := -K_D \dot{e} - K_p e = -K \underline{\varepsilon}$$

Con $K = [K_p \vdots K_D]$

$$K_D = \begin{bmatrix} k_{D1} & & \\ & \ddots & \\ & & k_{Dn} \end{bmatrix}$$

$$K_p = \begin{bmatrix} k_{p1} & & \\ & \ddots & \\ & & k_{pn} \end{bmatrix}$$

$$\begin{cases} F = G = H = 0 \\ L = -K \end{cases}$$

SISTEMA CONTROLLATO

$$\dot{\underline{\varepsilon}} = A_c \underline{\varepsilon}$$

con $A_c = \begin{bmatrix} 0 & I_n \\ -K_p & -K_D \end{bmatrix}$

$$\boxed{\ddot{e}(t) + K_D \dot{e}(t) + K_p e(t) = 0}$$

$$e(0) \neq 0$$
$$\dot{e}(0) \neq 0$$