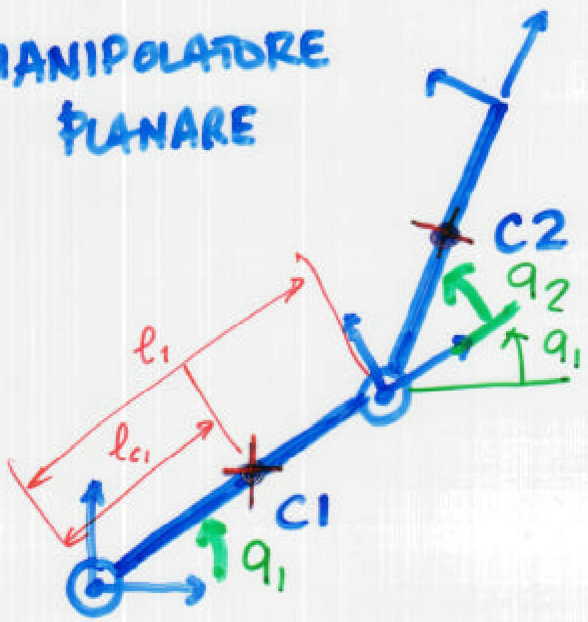
MANIPOLATORE PLANARE



$C = C_1 + C_2$

$P = P_1 + P_2$

$\ell_{c_1} \quad \ell_{c_2}$

$\ell_1 \quad \ell_2$

p_{c_1} ; p_{c_2} ; $\dot{p}_{c_1}$; $\dot{p}_{c_2}$

$$p_{c_1} = \begin{bmatrix} \ell_{c_1} c_1 \\ \ell_{c_1} s_1 \\ 0 \end{bmatrix} \qquad \dot{p}_{c_1} = \begin{bmatrix} -\ell_{c_1} s_1 \dot{q}_1 \\ \ell_{c_1} c_1 \dot{q}_1 \\ 0 \end{bmatrix} = \begin{bmatrix} \vdots \\ 0 \end{bmatrix} \dot{q}_1$$

$$\underline{\alpha} = \begin{bmatrix} q_1 + q_2 \\ 0 \\ 0 \end{bmatrix}$$ TRASCURIAMO L'ASSETTO

$$J_{L1}^{(1)} = \begin{bmatrix} -\ell_{c_1} s_1 \\ \ell_{c_1} c_1 \\ 0 \end{bmatrix} \qquad J_{L2}^{(1)} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$J_L^{(1)} = \begin{bmatrix} -\ell_{c_1} s_1 & 0 \\ \ell_{c_1} c_1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$p_{c_2} = \begin{bmatrix} \ell_1 c_1 + \ell_{c_2} c_{12} \\ \ell_1 s_1 + \ell_{c_2} s_{12} \\ 0 \end{bmatrix} \qquad v_{ci} \equiv \dot{p}_{ci}$$

$$\dot{p}_{c_2} = \begin{bmatrix} -\ell_1 s_1 \dot{q}_1 - \ell_{c_2} s_{12}(\dot{q}_1 + \dot{q}_2) \\ \ell_1 c_1 \dot{q}_1 + \ell_{c_2} c_{12}(\dot{q}_1 + \dot{q}_2) \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \quad J_L^{(2)} \quad \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \left[\begin{array}{c:c} -\ell_1 s_1 - \ell_{c_2} s_{12} & -\ell_{c_2} s_{12} \\ \ell_1 c_1 + \ell_{c_2} c_{12} & \ell_{c_2} c_{12} \\ 0 & 0 \end{array}\right]$$

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$$C_i = \frac{1}{2}\left[m_i \, \underline{v}_{ci}^T \underline{v}_{ci} + \underline{\omega}_i^T \Gamma_i \underline{\omega}_i \right]$$

$$\Gamma_i = \begin{bmatrix} \boxed{\Gamma_{x_i}} & 0 & 0 \\ 0 & \boxed{\Gamma_{y_i}} & 0 \\ 0 & 0 & \Gamma_{z_i} \end{bmatrix}$$

$$H = H_1 + H_2$$

$$H_i = \left[J_L^{(1)T} m_1 J_L^{(1)} + J_A^{(1)T} \Gamma_1 J_A^{(1)} \right] \Rightarrow$$

$$\Leftarrow \frac{1}{2} m_i v_{ci}^T v_{ci} + \frac{1}{2} \underline{\omega}_i^T \Gamma_i \underline{\omega}_i$$

BRACCIO 1 —o—

$$v_{c_1}^T v_{c_1} = \left[-\ell_{c_1} s_1 \dot{q}_1 \;\vdots\; \ell_{c_1} c_1 \dot{q}_1 \;\vdots\; 0 \right] \begin{bmatrix} -\ell_{c_1} s_1 \dot{q}_1 \\ \ell_{c_1} c_1 \dot{q}_1 \\ 0 \end{bmatrix}$$

$$\ell_{c_1}^2 s_1^2 \dot{q}_1^2 + \ell_{c_1}^2 c_1^2 \dot{q}_1^2 = \ell_{c_1}^2 \dot{q}_1^2$$

$\frac{1}{2} m_1 \ell_{c_1}^2 \dot{q}_1^2$ EN. CIN Braccio 1

$$\Downarrow$$

$$\frac{1}{2} \dot{q}_1 \left[m_1 \ell_{c_1}^2 \right] \dot{q}_1$$

$$\frac{1}{2}\underline{\omega}_i^T \Gamma \underline{\omega}_i$$

$$\underline{\omega}_i \ ? \begin{bmatrix} 0 \\ 0 \\ \dot{q}_{(1)} \end{bmatrix}$$

$$\frac{1}{2}\begin{bmatrix} 0 & 0 & \dot{q}_1 \end{bmatrix} \begin{bmatrix} \boxed{\Gamma_{x1}} & & \\ & \boxed{\Gamma_{y1}} & \\ & & \boxed{\Gamma_{z1}} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{q}_1 \end{bmatrix}$$

$$\frac{1}{2}\begin{bmatrix} 0 & 0 & \dot{q}_1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \Gamma_{z1}\dot{q}_1 \end{bmatrix} = \frac{1}{2}\Gamma_{z1}\dot{q}_1^2$$

$$= \frac{1}{2}\dot{q}_1 \boxed{\Gamma_{z1}} \dot{q}_1$$

$$C_1 = \frac{1}{2}\dot{q}_1 \left[m_1 \ell_{c_1}^2 + \Gamma_{z1} \right] \dot{q}_1$$

$$\Downarrow$$

$$\frac{1}{2}\underline{\dot{q}}^T H_i \underline{\dot{q}} \ ?$$

$$\frac{1}{2}\begin{bmatrix}\dot q_1 & \dot q_2\end{bmatrix}\underbrace{\begin{bmatrix} m_1 \ell_{c_1}^2 + I_{z_1} & 0 \\ 0 & 0\end{bmatrix}}_{H_1 = H_{1L} + H_{1A}}\begin{bmatrix}\dot q_1 \\ \dot q_2\end{bmatrix}$$

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BRACCIO 2

$$v_{c_2}^T v_c^2 =$$

$$\Big[\overbrace{-\ell_1 s_1 \dot q_1 - \ell_{c_2} s_{12}(\dot q_1 + \dot q_2)}^{A} \; \vdots \; \underbrace{\ell_1 c_1 \dot q_1 + \ell_{c_2} c_{12}(\dot q_1 + \dot q_2)}_{B} \; \vdots \; 0\Big]\begin{bmatrix} A \\ B \\ 0 \end{bmatrix}$$

$$A^2 + B^2$$

$$\ell_1^2 s_1^2 \dot q_1^2 + \ell_{c_2}^2 s_{12}^2 (\dot q_1 + \dot q_2)^2 + 2\ell_1 s_1 \ell_{c_2} s_{12} (\dot q_1 + \dot q_2)\dot q_1$$

$$+$$

$$\ell_1^2 c_1^2 \dot q_1^2 + \ell_{c_2}^2 c_{12}^2 (\dot q_1 + \dot q_2)^2 + 2\ell_1 c_1 \ell_{c_2} c_{12} (\dot q_1 + \dot q_2)\dot q_1$$

$$\ell_1^2 \dot{q}_1^2 + \ell_{c_2}^2 (\dot{q}_1 + \dot{q}_2)^2$$

$$+ 2\ell_1 \ell_{c_2} \left[\dot{q}_1 + \dot{q}_2\right] \dot{q}_1 \left[\underbrace{s_1 s_{12} + c_1 c_{12}}_{c_2}\right] = [\bullet]$$

$$\frac{1}{2} m_2 [\bullet] \rightsquigarrow \frac{1}{2} \underline{\dot{q}}^T \left[\; H_{2L} \;\right] \underline{\dot{q}}$$

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$$\frac{1}{2} \underline{\omega}_2^T \, I_2 \, \underline{\omega}_2 \qquad \underline{\omega}_2 = \begin{bmatrix} 0 \\ 0 \\ \dot{q}_1 + \dot{q}_2 \end{bmatrix}$$

$$\frac{1}{2} \left[\dot{q}_1 + \dot{q}_2\right] I_{zz} \left[\dot{q}_1 + \dot{q}_2\right]$$

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$$\rightsquigarrow H_2 \; ?!?$$

$$\frac{1}{2} \begin{bmatrix} \dot{q}_1 & \dot{q}_2 \end{bmatrix} \begin{bmatrix} \quad ? \quad \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

$$\Gamma_{22}(\dot q_1+\dot q_2)^2 = \Gamma_{22}\left[\dot q_1^2+\dot q_2^2+2q_1q_2\right]$$

$$[\dot q_1 \quad \dot q_2]\boxed{\begin{bmatrix}\Gamma_{22} & \Gamma_{22}\\ \Gamma_{22} & \Gamma_{22}\end{bmatrix}}\begin{bmatrix}\dot q_1\\ \dot q_2\end{bmatrix} \xrightarrow{\equiv} H_{2A}$$

$$[\dot q_1 \quad \dot q_2]\begin{bmatrix}\Gamma_{22}\dot q_1+\Gamma_{22}\dot q_2\\ \Gamma_2\dot q_1+\Gamma_2\dot q_2\end{bmatrix} = \Gamma_2\dot q_1^2+\Gamma_2\dot q_1\dot q_2+\Gamma_2 q_1q_2+\Gamma_2 q_2^2$$ OK!

data M non simmetrica

$$M = M_S + M_A$$

$$\left.\begin{aligned} M_S &= \text{SIMMETRICA} = \frac{M+M^T}{2}\\ M_A &= \text{ANTISIM} = \frac{M-M^T}{2}\end{aligned}\right\} \frac{M+M^T+M-M^T}{2}$$

$$= \frac{2M}{2} = M$$

H_{2L} ?

$$m_2 \begin{bmatrix} \ell_1^2 + \ell_{c_2}^2 + 2\ell_1 \ell_{c_2} c_2 & \ell_{c_2}^2 + \ell_1 \ell_{c_2} c_2 \\ \ell_{c_2}^2 + \ell_1 \ell_{c_2} c_2 & \ell_{c_2}^2 \end{bmatrix}$$

termine fuori dalla diagonale :

$$2\ell_{c_2}^2 + 2\ell_1 \ell_{c_2} c_2 \begin{cases} \rightarrow (1,2) \\ \rightarrow (2,1) \end{cases}$$

H(t) del manipolatore =

$= H_1(t) + H_{2L}(t) + H_{2A}(t)$

pag 119 in alto

$H\underline{\ddot{q}} + C\underline{\dot{q}} + \underline{g} + \tau$

OK

SE ROBOT È PLANARE $\Rightarrow 0$

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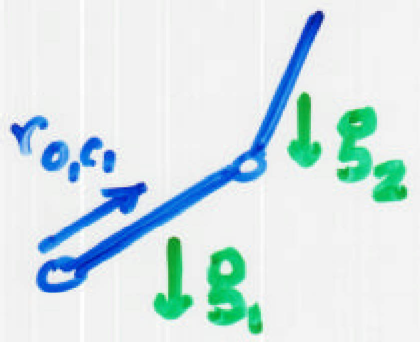
IPOTESI { NON PLANARE



$\gamma = \begin{bmatrix} 0 \\ -g \\ 0 \end{bmatrix} \equiv -9.81$

$\underline{g}_1 \Rightarrow \quad \mathcal{P} = \mathcal{P}_1 + \mathcal{P}_2$

Calcoliamo $\mathcal{P}_1 = -m_1 \gamma^T r_{0,c_1}$

$\underline{r}_{0,c_1} \equiv p_{c_1} = \begin{bmatrix} \ell_{c_1} c_1 \\ \ell_{c_1} s_1 \\ 0 \end{bmatrix}$

$\mathcal{P}_1 = m_1 g\, \ell_{c_1} s_1$

$\mathcal{P}_2 = m_2 g\, \ell_1 s_1 + \ell_{c_2} s_{12}$

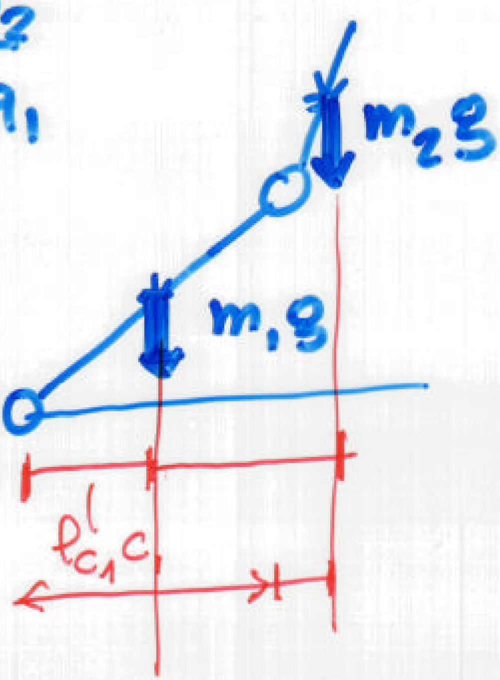
EQ LAGRANGE

$\ldots\ldots \quad -\dfrac{\partial \mathcal{P}}{\partial q}$

$$-\frac{\partial \mathcal{P}}{\partial q_1} = -\frac{\partial \mathcal{P}_1}{\partial q_1} - \frac{\partial \mathcal{P}_2}{\partial q_1}$$

$$\downarrow$$

$$m_1 g \ell_{c_1} c_1$$



CALCOLO DI C