FUNZIONI CINEMATICHE DIRETTE & INVERSE DI POSIZIONE & VELOCITA' $\boxed{4}$

RISULTATI — o — AZIONI

CARTESIANO

$$\underset{6\times1}{\underline{p}} = \begin{bmatrix} \underline{x} \\ \cdots \\ \underline{\alpha} \end{bmatrix} \begin{matrix} 3\times1 \\ \\ 3\times1 \end{matrix}$$

GIUNTI

$$\underline{q} = \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_6 \end{bmatrix}$$

F.C. DIRETTA POSIZIONE

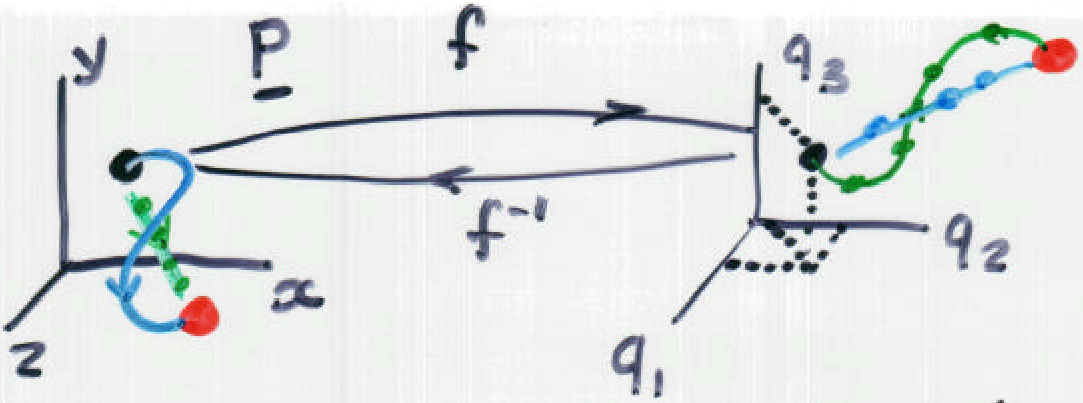
PDKF

$$\underline{p} = \underline{f}(\underline{q})$$

NON LINEARI

INVERSA

$$\underline{q} = \underline{f}^{-1}(\underline{p})$$



$k_1 \quad k_2$

v_A ● $\equiv \underline{x}_A$

v_B ● $\equiv \underline{x}_B$

$\underline{x}_A \lambda + (1-\lambda)\underline{x}_B$

$k_1 + k_2 = 1$

$0 \le \lambda \le 1$

— o —

VELOCITÀ

$$\underline{p} \rightarrow \dot{\underline{p}} = \frac{dp}{dt} \rightarrow \dot{p} = \begin{bmatrix} \dot{\underline{x}} \\ \cdots \\ \dot{\underline{\alpha}} \end{bmatrix} \quad \underline{v} \quad !!!$$

$$\underline{q} \rightarrow \dot{\underline{q}} = \frac{dq}{dt} \rightarrow \dot{q} = \begin{bmatrix} \dot{q}_1 \\ \vdots \\ \dot{q}_6 \end{bmatrix}$$

S.I.

Km Kg k minuscolo

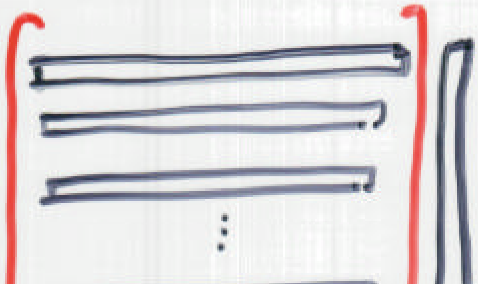
s~~e~~c s

$$\underline{p} = \underline{f}(\underline{q}) \cdot \begin{cases} p_1 = f_1(\underline{q}) \\ \vdots \\ p_6 = f_6(\underline{q}) \end{cases}$$

$$\dot{p}_1 = \frac{dp_1}{dt} = \frac{\partial f_1}{\partial q_1}\frac{dq_1}{dt} + \frac{\partial f_1}{\partial q_2}\frac{dq_2}{dt} \dots$$

$$\dot{p}_6 = \frac{\partial f_6}{\partial q_1}\frac{dq_1}{dt} + \dots$$

$$\dot{p}_i = \begin{bmatrix} \frac{\partial f_i}{\partial q_1} & \frac{\partial f_i}{\partial q_2} & \dots & \frac{\partial f_i}{\partial q_6} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \vdots \\ \dot{q}_6 \end{bmatrix}$$

$\dot{\underline{p}} =$ 

$J(\underline{q}(t))$

jacobiano 6×6 nello spazio 3D

3×3 nel piano

$\dot{\underline{p}} = J(\underline{q}(t))\,\dot{\underline{q}}$ CIN DIR VEL.

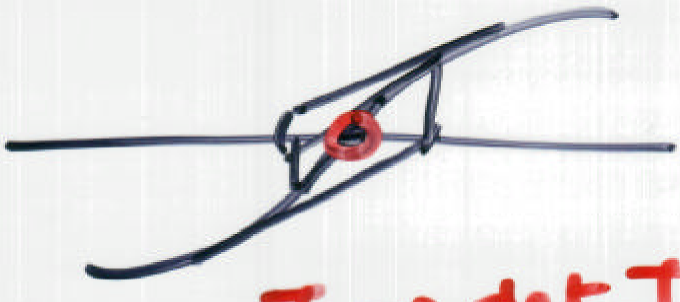
$\dot{\underline{q}} = J^{-1}(\underline{q}(t))\,\dot{\underline{p}}$ CIN INV VEL.

$\underline{p}(t) = \underline{f}(\underline{q}(t))$

$\underline{q}(t) = \underline{f}^{-1}(\underline{p}(t))$

$\dot{\underline{p}}(t) = J(\underline{q}(t))\,\dot{\underline{q}}(t)$

$\dot{\underline{q}}(t) = J^{-1}(\underline{q}(t))\,\dot{\underline{p}}(t) \leftarrow$

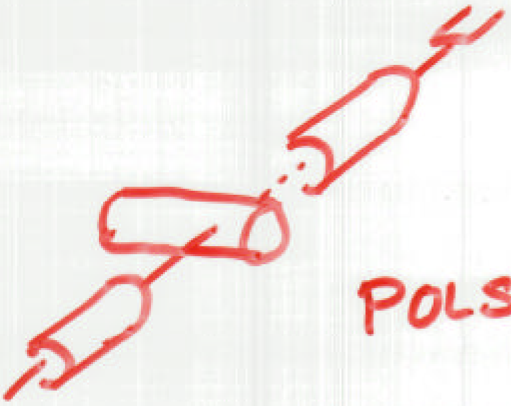


$J \to \det J = 0$ per certe $\underline{q}^{*}(t)$

SINGOLARITÀ CINEMATICA $\rightarrow \underline{q}^*(t)$

$\downarrow$

CONFIGURAZ. SINGOLARE



POLSO SINGOLARE!

MOLTO COMUNE ∞

$\dot{p} = J\dot{q} \qquad \dot{q} = J^{-1}\dot{p}$

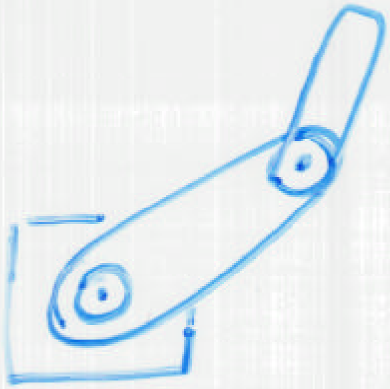
$\Downarrow$

$$\dot{q} = \frac{1}{\det J}\left[X\right]\dot{p}$$

$\det J \to 0 \qquad \dot{q} \to \infty$

$\Delta q \cong J^{-1}\Delta p$

$\Delta q \to \infty$



CIN POSIZIONE

$$\underline{p}(t) = \underline{f}(\underline{q}(t)) \quad ?!$$

- APPLICARE CONV. D.H.
- T_i^{i-1} matrici omogenee

$$\left.\begin{aligned} T_1^0 &= T_1^0(q_1) \\ T_2^1 &= T_2^1(q_2) \\ &\vdots \\ T_6^5 &= T_6^5(q_6) \end{aligned}\right\} T_6^0 = T_6^0(q_1, q_2, \dots, q_6)$$

$$\mathbf{T} \equiv \left[\begin{array}{c|c} \mathbf{R} & \underline{t} \\ \hline 0\ 0\ 0 & 1 \end{array}\right]$$

$T_6^0 \equiv T$ è quello che
mi interessa
studiare con la cinematica

$\underline{P} = \begin{bmatrix} \underline{x} \\ \underline{\alpha} \end{bmatrix}$

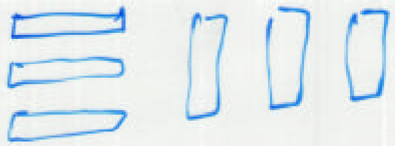
$$\left[\begin{array}{c|c} R & t \\ \hline 0^T & 1 \end{array}\right]$$

$\equiv \underline{x}$

NASCONDE $\underline{\alpha}$
AL SUO INTERNO

$R^T R = I$

— o —



$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

— o —

ANGOLI DI EULERO

ANGOLI DI ROLL - PITCH - YAW

RPY : $\theta_x \quad \theta_y \quad \theta_z$

$$\left.\begin{array}{l} Rot(\underline{i}, \theta_x) \\ Rot(\underline{j}, \theta_y) \\ Rot(\underline{k}, \theta_z) \end{array}\right\} \begin{array}{l} R_x \\ R_y \\ R_z \end{array} \rightarrow R_{roll} = R_z R_y R_x$$

Pitch
Yaw

$$\begin{bmatrix} R & t \\ 0^T & 1 \end{bmatrix}$$

α_1	α_2	α_3	
ϕ,	θ,	ψ	?
θ_x	θ_y	θ_z	?

$$\underline{p} = \begin{bmatrix} \underline{x} \\ \underline{\alpha} \end{bmatrix} \begin{array}{l} = t \\ = ? \end{array}$$

DIPENDE DA QUALI ANGOLI ABBIAMO SCELTO