JACOBIANO ANALITICO E JACOBIANO GEOMETRICO

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$$\underline{p} = \begin{bmatrix} \underline{x} \\ \underline{\alpha} \end{bmatrix} \qquad \dot{\underline{p}} = \begin{bmatrix} \dot{\underline{x}} \equiv \underline{v} \\ \dot{\underline{\alpha}} \equiv \underline{\omega} \end{bmatrix}$$

$$\underline{p} = \underline{f}(\underline{q}) \qquad \dot{p} = J(q(t))\,\dot{q}$$

$$\dot{x} = J_L \dot{q} \longleftarrow$$

$$\dot{\alpha} = J_A \dot{q} \longleftarrow \qquad J = \begin{bmatrix} \boxed{3 \times 6} \\ \boxed{3 \times 6} \end{bmatrix}$$

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$\alpha \equiv \alpha_E$ ANGOLI DI EULERO

$$\dot{\underline{\alpha}}_E \equiv \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \qquad \underline{b}_\phi \quad \underline{b}_\theta \quad \underline{b}_\psi$$

$$\underline{\omega} = \underline{b}_\phi \dot{\phi} + \underline{b}_\theta \dot{\theta} + \underline{b}_\psi \dot{\psi}$$

$$\underline{b}_\phi = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad \underline{b}_\theta = \begin{bmatrix} \cos\phi(t) \\ \sin\phi(t) \\ 0 \end{bmatrix}$$

$$\underline{b}_\psi = \begin{bmatrix} \sin\phi(t) \cdot \sin\theta(t) \\ -\cos\phi(t) \cdot \sin\theta(t) \\ \cos\theta(t) \end{bmatrix}$$

$$M_E = \begin{bmatrix} \underline{b}_\phi & \underline{b}_\theta & \underline{b}_\psi \end{bmatrix}$$

$$\underline{\omega} = M_E \, \dot{\alpha}_E \qquad \boxed{\begin{matrix} \phi = 90^\circ \\ \theta = 0^\circ \end{matrix}}$$

$$M_E(\cdot) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \qquad \cancel{\tilde{M}_E} =$$

$$\boxed{\text{RPY}}$$

$\theta_x \ \theta_y \ \theta_z$ piccoli

$$M_{RPY} = \begin{bmatrix} c_z c_y & -s_z & 0 \\ s_z c_y & c_z & 0 \\ -s_y & 0 & 1 \end{bmatrix} \simeq \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\tilde{M}_E = \begin{bmatrix} \sin\psi \sin\theta & \cos\psi & 0 \\ \cos\psi \sin\theta & -\sin\psi & 0 \\ \cos\theta & 0 & 1 \end{bmatrix}$$

JACOBIANO GEOMETRICO

$\dot{\alpha} \equiv \omega$

$$\dot{p} = \begin{bmatrix} \underline{v} \\ \underline{\omega} \end{bmatrix} = \begin{cases} \to \boxed{3\times 6 \ldots} \\ \to \boxed{3\times 6} \end{cases} \qquad \dot{q}$$

$$\underline{v} = J_{L1}\,\dot{q}_1 + \cdots + J_{L6}\,\dot{q}_6$$

$$\underline{\omega} = J_{A1}\,\dot{q}_1 + \cdots + J_{A6}\,\dot{q}_6$$

$q_i\,;\,\dot{q}_i$ giunto i <u>**PRISMATICO**</u>

$$J_{Li}\,\dot{q}_i = \underline{k}_{i-1}\,\dot{q}_i$$

$$J_{Li} = \left[\underline{k}_{i-1}\right]_{\mathcal{R}_0} \equiv R^0_{i-1} \text{ prendiamo la } 3^a \text{ colonna}$$

J_{Ai}

$$J_{Ai}\dot{q}_i = \underline{0}$$

$$J_{Ai} = \underline{0}$$

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i è giunto rotoidale

$$J_{Ai}\dot{q}_i = \underline{k}_{i-1}\dot{q}_i$$

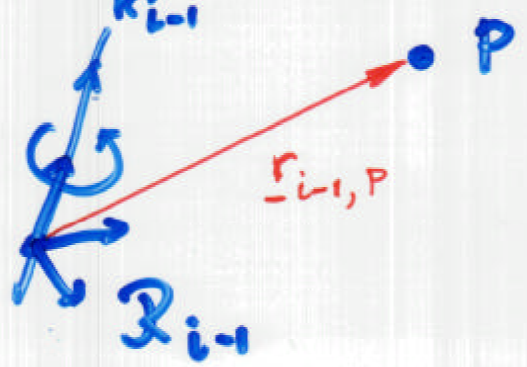
$$J_{Ai} = \left[\underline{k}_{i-1}\right]_{\mathcal{R}_0} \qquad \wedge$$

$$J_{Li}\dot{q}_i = (\underline{k}_{i-1} \times \underline{r}_{i-1,P})\dot{q}_i$$

$$\downarrow$$

$$\left[k_{i-1}\right]_{\mathcal{R}_0} \times \left[\underline{r}_{i-1,P}\right]_{\mathcal{R}_0}$$

$\underline{r}_{i-1,P}$ VETTORE DALL'ORIGINE DI $\mathcal{R}_{i-1}$ ALLA "PUNTA" DEL MANIPOLATORE



$$[k_{i-1}]_{\mathcal{R}_0} \times [\underline{r}_{i-1,P}]_{\mathcal{R}_0}$$

$$R^0_{i-1}\left[\underbrace{[k_{i-1}]_{\mathcal{R}_{i-1}}}_{\begin{bmatrix}0\\0\\1\end{bmatrix}} \times [\underline{r}_{i-1,P}]_{\mathcal{R}_{i-1}}\right]$$

TIPO GIUNTO	J_{Li}	J_{Ai}
PRISMATICO	$\underline{k}_{i-1}$	0
ROTOIDALE	$\boxed{\underline{k}_{i-1} \times \underline{r}_{i-1,P}}$	$\underline{k}_{i-1}$

CINEMATICA VELOCITA'

$$\phi = q_1(t) \qquad \dot{\phi} = \dot{q}_1(t)$$

$$\theta = q_2(t) \qquad \dot{\theta} = \dot{q}_2(t)$$

$$\psi = 90 \qquad \dot{\psi} = 0$$

$$\left[\dot{\underline{\alpha}}\right] = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}}_{J} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

$$\begin{vmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{vmatrix} = -(\ell_3 s_1 c_2 + \ell_2 s_1)\dot{q}_1 - \ell_3 c_1 s_2 \dot{q}_2$$

$-\ell_3 c_1 s_2$

$$\left[-(\ell_3 s_1 c_2 + \ell_2 s_1) \quad\quad \right] \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

$$J_L = \begin{bmatrix} -(\ell_3 s_1 c_2 + \ell_2 s_1 & -\ell_3 c_1 s_2 \\ (\ell_3 c_1 c_2 + \ell_2 c_1) & -\ell_3 s_1 s_2 \\ 0 & \ell_3 c_2 \end{bmatrix}$$

$$J_A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$