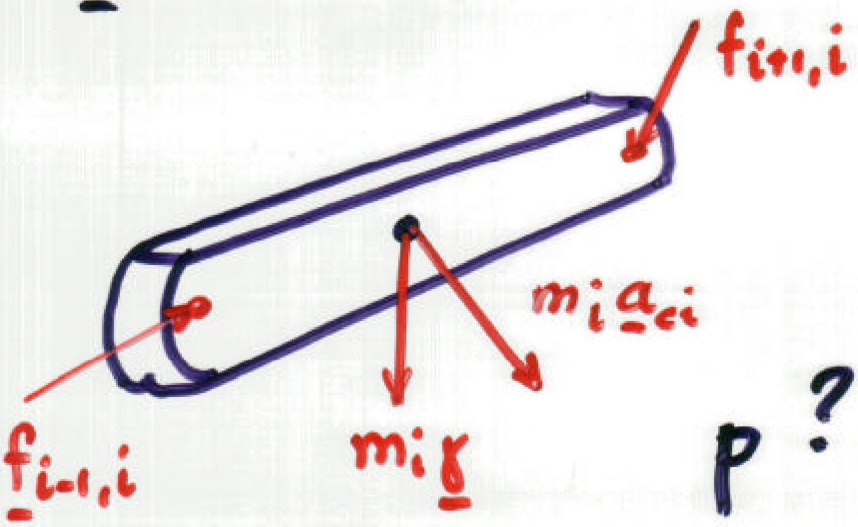
EQUILIBRIO FORZE (p.110)

$$\underline{f}_{i-1,i} + \underline{f}_{i+1,i} + m_i \underline{\gamma} - m_i \underline{a}_{ci} = \underline{0}$$



$$p \overset{?}{=} p_{ci}$$

$$\dot{p} = J\dot{q} \rightarrow \ddot{p} = \dot{J}\dot{q} + J\ddot{q}$$

DINAMICA è funzione delle $q_i(t)$

$$a_{ci} = a_{ci}(\underline{q})$$

EQUILIBRIO MOMENTI

in $R_i \equiv R_0$

$$\underline{N}_{i-1,i} + \underline{N}_{i+1,i} + \underline{r}_{ci,i-1} \times \underline{f}_{i-1,i}$$

$$+ r_{ci,i} \times f_{i+1,i}$$

$$\frac{d}{dt}\left[\Gamma \underline{\omega}\right] \cdots \quad - \Gamma_i \underline{\dot{\omega}}_i - \underline{\omega}_i \times \Gamma_i \underline{\omega}_i = \underline{0}$$

$\underline{\dot{\omega}}_i$ = accelerazione angolare del braccio i

$\underline{\omega}_i$ = velocità angolare del braccio i

$\Gamma_i = \left[\; 3 \times 3 \;\right]$ matrice d'inerzia del braccio i rispetto a C_i

OKKIO! → il tutto va rappresentato in R_0

EQUAZIONI DI LAGRANGE

CONCETTO DI ENERGIA

- ENERGIA CINETICA
- " POTENZIALE

FUNZIONE (ENERGIA) DI LAGRANGE

FUNZIONE DI STATO

$$\mathcal{L}(\underline{q},\underline{\dot{q}}) = \mathcal{C}(\underline{q},\underline{\dot{q}}) - \mathcal{P}(q)$$

E. CINETICA TOTALE

E. POTENZIALE TOTALE

$$\mathcal{C}(q,\dot{q}) = \sum_{i=1}^{6} C_i(\underline{q},\underline{\dot{q}})$$

$$\mathcal{P}(\underline{q}) = \sum_{i=1}^{6} \mathcal{P}_i(q)$$

BRACCIO i-esimo

$$\frac{d}{dt}\left[\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right] - \frac{\partial \mathcal{L}}{\partial q_i} = \mathcal{T}_i$$

FORZA GENERALIZZATA i-esima

$i = 1, \dots, 6$

6 EQUAZIONI SCALARI DIFFERENZIALI 2° ORDINE

$$\Downarrow$$

EQUAZIONE (MODELLO DINAMICO) DEL MANIPOLATORE

$$\underline{H}(\underline{q})\,\underline{\ddot{q}}(t) + C(\underline{q}, \underline{\dot{q}})\,\dot{q}(t) + \underline{g}(\underline{q}) = \underline{\tau}$$

$$\frac{d}{dt}\left[\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right] - \frac{\partial \mathcal{L}}{\partial q_i} = \mathcal{F}_i$$

↓

attrito + altro

↓

$$-\frac{\partial \mathcal{D}}{\partial \dot{q}_i}$$

$$\frac{d}{dt}\left[\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right] - \frac{\partial \mathcal{L}}{\partial q_i} + \frac{\partial \mathcal{D}}{\partial \dot{q}_i} = \mathcal{F}_i^{na}$$

$C_i(\underline{q},\underline{\dot{q}})$ = EN. CIN. VELOCITA LINEARE

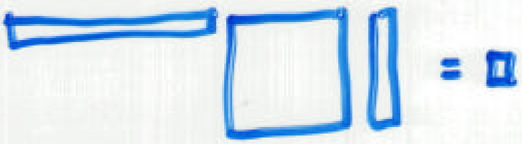
+

" " VELOCITÀ ANGOLARE

=

$\frac{1}{2}\,\underline{v}^T_{c_i}(\underline{q},\underline{\dot{q}})\, m_i\, \underline{v}_{c_i}(\underline{q},\underline{\dot{q}})$ 1° TERMINE

$$\frac{1}{2}\,\underline{\omega}_i^T(\underline{q},\underline{\dot{q}})\,\Gamma_{0i}\,\underline{\omega}_i(\underline{q},\underline{\dot{q}})$$

$\frac{1}{2}$ 

$$\underline{x}^T M \underline{x}$$

FORMA QUADRATICA

$$\begin{bmatrix}\underline{v}_{ci}\\ \underline{\omega}_i\end{bmatrix} = \underline{\dot{p}}_{ci} = J_{ci}\,\underline{\dot{q}} = \left[\begin{array}{cccc}\square & \square & \square \cdots & \square\\ \hline \square & \square & \square \cdots & \square\end{array}\right]\underline{\dot{q}}$$

$$\underline{v}_{ci} = J_{L1}^{(i)}\,\dot{q}_1 + J_{L2}^{(i)}\,\dot{q}_2 + \cdots$$

$$\underline{\omega}_i = J_{A1}^{(i)}\,\dot{q}_1 + \cdots \text{ ecc.}$$

$$J_L^{(i)} = \left[J_{L1}^{(i)} \quad J_{L2}^{(i)} \quad J_{Li}^{(i)} \cdots \underline{0} \quad \underline{0}\right]$$

$$J_A^{(i)} = \left[J_{A1}^{(i)} \quad J_{A2}^{(i)} \quad J_{Ai}^{(i)} \cdots \underline{0} \quad \underline{0}\right]$$

$$C = \sum_i C_i = \frac{1}{2}\sum_i\left[\underline{v}_{ci}^T m_i \underline{v}_{ci} + \underline{\omega}_i^T \Gamma_{0i}\,\underline{\omega}_i\right]$$

$$= \frac{1}{2}\sum_i \Big[\dot{\underline{q}}^T \boxed{J_L^{(i)T} m_i J_L^{(i)}} \dot{\underline{q}} + $$

$$+ \dot{\underline{q}}^T \boxed{J_A^{(i)T} \Gamma_i J_A^{(i)}} \dot{\underline{q}} \Big]$$

$$= \frac{1}{2} \dot{\underline{q}}^T \boxed{\sum_i \Big[J_L^{(i)T} m_i J_L^{(i)} + J_A^{(i)T} \Gamma_i J_A^{(i)} \Big]} \dot{\underline{q}}$$

$$\Downarrow$$

$$\sum_i H_i(\underline{q})$$

$$= \frac{1}{2} \dot{q}^T H(q) \dot{q}$$