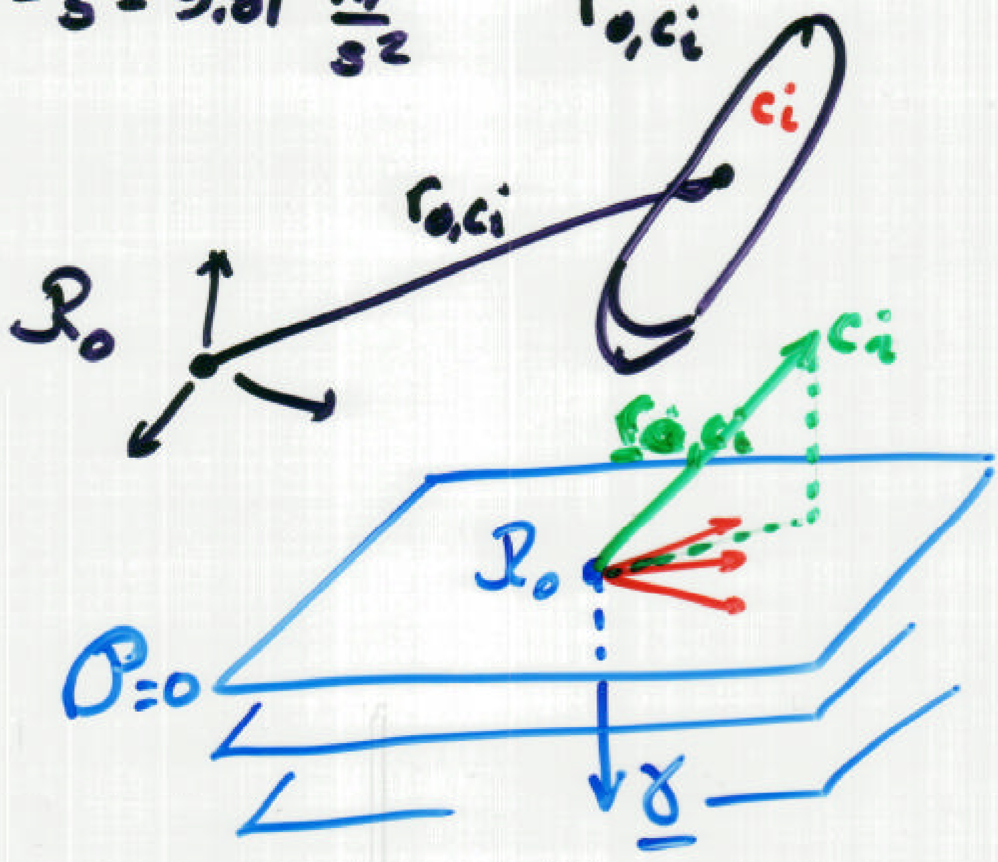
$$\mathcal{L} = \mathcal{C} - \mathcal{P}$$

ENERGIA POTENZIALE

$$\mathcal{P} = \sum_i \mathcal{P}_i$$

$$\boxed{\mathcal{P}_i = -m_i \underline{\gamma}^T \underline{r}_{0,c_i}(\underline{q})}$$

$$\|\gamma\| = g \cong 9.81 \ \frac{m}{s^2}$$



$$\mathcal{L} = C - P$$

$$\frac{d}{dt}\left[\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right] - \frac{\partial \mathcal{L}}{\partial q_i} + \frac{\partial D}{\partial \dot{q}_i} = \mathcal{F}_i$$

$$D = \sum_i D_i = \frac{1}{2}\,\underline{\dot{q}}^T B\, \underline{\dot{q}}$$

$$\begin{bmatrix} \beta_1 & & \emptyset \\ & \beta_2 \ddots & \\ \emptyset & & \beta_6 \end{bmatrix}$$

$\mathcal{F}_i = \tau_i$ motrici

$+\ \tau_{ei}$ dell'environment

(5.36)

$$\mathcal{F}_i = \begin{bmatrix} \mathcal{F}_1 \\ \vdots \\ \mathcal{F}_6 \end{bmatrix} = \tau_c + J^T F_e$$

$$\frac{d}{dt}\left[\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right] = \frac{d}{dt}\left[\frac{\partial C - \partial P}{\partial \dot{q}_i}\right]$$

$= 0$

$C(q, \dot{q})$

$P(q)$

$$= \frac{d}{dt}\frac{\partial C}{\partial \dot{q}_i} = \frac{d}{dt}\left[\sum_j H_{ij}\,\dot{q}_j\right]$$

$$= \sum_j H_{ij}\,\ddot{q}_j + \sum_j \frac{dH_{ij}}{dt}\,\dot{q}_j$$

$$\frac{d}{dt}\left[\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right] = \quad \cdot \quad \cdot \quad \cdot$$

$$= \sum_j H_{ij}(\underline{q})\,\ddot{q}_j + \sum_j \sum_k \frac{\partial H_{ij}}{\partial q_k}\,\dot{q}_k\,\dot{q}_j$$

$$\frac{d}{dt}\left[\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right] - \frac{\partial \mathcal{L}}{\partial q_i} = \mathcal{F}_i$$

$$\Downarrow$$

$$\sum_j H_{ij}(\underline{q})\ddot{q}_j + \sum_j \underbrace{\left[\sum_k h_{ijk}(\underline{q})\dot{q}_k\right]}_{c_{ij}(\underline{q},\underline{\dot{q}})} \dot{q}_j + g_i(q) = \mathcal{F}_i$$

$$\sum_j H_{ij}\ddot{q}_j + \sum_j c_{ij}(\ )\dot{q}_j + g_i = \mathcal{F}_i$$

i-esima equazione

SISTEMA $i = 1, \dots, 6$

$$H(\underline{q})\underline{\ddot{q}}(t) + C(\underline{q},\underline{\dot{q}})\underline{\dot{q}} + \underline{g}(\underline{q}) = \underline{\tau}$$

+ ATTRITO $B\dot{q}$

$$\underbrace{\sum_j H_{ij}\ddot{q}_j}_{H_{ii}\ddot{q}_i + \sum_{j\neq i} H_{ij}\ddot{q}_j} + \underbrace{\sum_j\sum_k h_{ijk}\dot{q}_k\dot{q}_j}_{\sum_j h_{ijj}\dot{q}^2 + \sum_j\sum_{k\neq j} h_{ijk}\dot{q}_k\dot{q}_j} + \underbrace{g_i}_{E} = \tau_i$$

$$\underbrace{H_{ii}\ddot{q}_i}_{A} + \underbrace{\sum_{j\neq i} H_{ij}\ddot{q}_j}_{B}$$

$$\underbrace{\sum_j h_{ijj}\dot{q}^2}_{C} + \underbrace{\sum_j\sum_{k\neq j} h_{ijk}\dot{q}_k\dot{q}_j}_{D}$$

A Inerzia equivalente sentita quando accelera $\ddot{q}_i$

B effetto inerziale dovuto agli altri bracci

E contributo gravitazionale

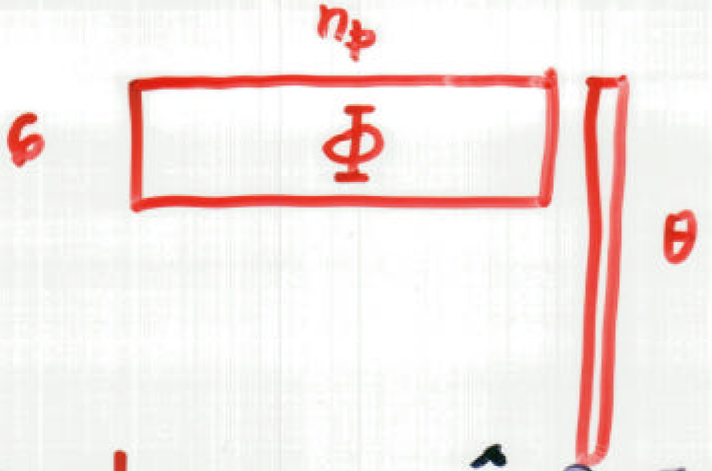
C effetto delle accelerazioni centrifughe

D effetto dovuto ai termini di Coriolis

$\boxed{\text{CAP } 2.11}$

$$H(\underline{q})\,\underline{\ddot{q}}(t) + C(q,\dot{q})\,\underline{\dot{q}}(t) + \underline{g}(\underline{q}) = \tau(t)$$

$$\dots = \underline{\Phi}(\ddot{q}(t), \dot{q}(t), q(t))\,\underline{\theta} = \tau$$



$$A\underline{x} = \underline{b} \longrightarrow \underline{\hat{x}} \; ? = (A^T A)^{-1} A^T \underline{b}$$

$$\Phi\theta = \tau$$

$$Ax - b = 0$$
$$Ax - b = e$$

MATRICE DI REGRESSIONE

SI PUÒ RICAVARE $\underline{\theta}$

$$\underline{\dot{H}} - 2C = N(\underline{q}, \underline{\dot{q}})$$

$\underline{N}$ è antisimmetrica

ANTISIMMETRICA

$$0 \equiv \underline{x}^T S \underline{x}$$ FORMA QUADRATICA

$$\underline{x}^T M \underline{x}$$

—o—

COME SI FA A SCRIVERE UN'EQUAZIONE DI STATO DEL MANIPOLATORE

$$H\ddot{q} + C\dot{q} + g = \tau$$

?

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

SE LINEARE

OPPURE

$$\begin{cases} \dot{x} = f(x, u) \\ y = g(x, u) \end{cases}$$

$$\underline{x} = \begin{bmatrix} q \\ \dot{q} \end{bmatrix} = \begin{bmatrix} \underline{x}_1 \\ \underline{x}_2 \end{bmatrix} \qquad \dot{\underline{x}} = \begin{bmatrix} \dot{\underline{x}}_1 \\ \dot{\underline{x}}_2 \end{bmatrix}$$

12 STATI

$$\dot{x} = \begin{bmatrix} \dot{q} \\ \ddot{q} \end{bmatrix} = \begin{bmatrix} \underline{x}_2 \\ \text{dalle equazioni di Lagrange} \end{bmatrix}$$

$$H\ddot{q} + C\dot{q} + g = \tau$$

$$\underline{\ddot{q}} = -H^{-1}C\underline{\dot{q}} - H^{-1}\underline{g} + H^{-1}\underline{\tau}$$

H^{-1} ESISTE SEMPRE

H è definita positiva