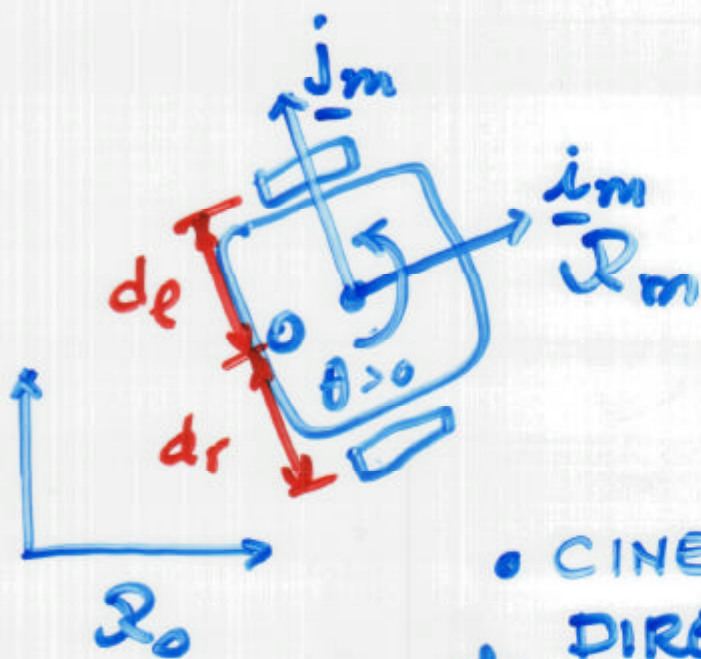




• CINEMATICA
& DIRETTA
INVERSA

r_r ; r_e raggi ruote

$r_r = r_e$ per ipotesi $= r$

$\|\omega_r\| = \dot{\varphi}_r(t)$
 $\|\omega_e\| = \dot{\varphi}_e(t)$ } variabili di comando
giunto

d_r ; d_e lunghezze semiasse
 $d_r = d_e = d$ per ipotesi

Z_m sta sull'asse tale che

Ruota destra è centrata in
ruota sinistra

$$\begin{bmatrix} 0 \\ d \\ 0 \end{bmatrix}_{Z_m}$$

$$\begin{bmatrix} 0 \\ d \\ 0 \end{bmatrix}_{Z_m}$$

VARIABILI GIUNTO

$$q_r(t) \text{ e } q_e(t)$$

$$\dot{q}_r = v_r(t) \quad v_e(t) = \dot{q}_e$$

Comandi dati come incremento di conteggi ?!?



ENCODER

KEPHERA II

24 slot
motoriduttore da 25:1

$$24 \times 25 = 600$$

$$\frac{600}{2\pi} = \text{risoluzione} \times \text{rad/rad}$$

$$t \rightarrow t_k; t_{k+1}; t_{k+2}$$

$$\begin{array}{l} T = 1 \text{ ms} \\ 10 \text{ ms} \\ 20 \text{ ms} \end{array}$$

$$v_z = r \dot{\phi}_r$$

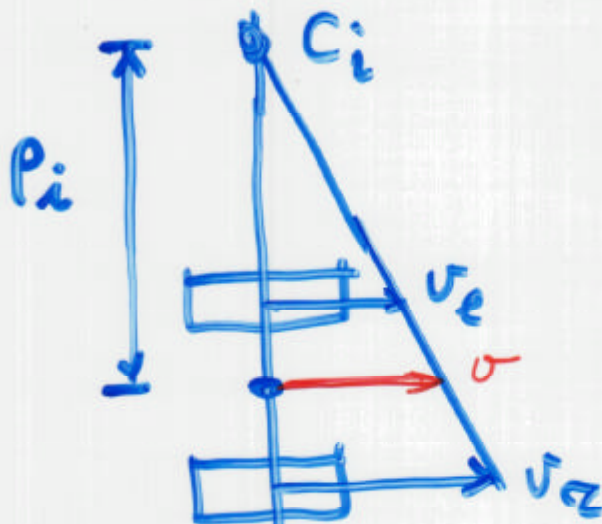
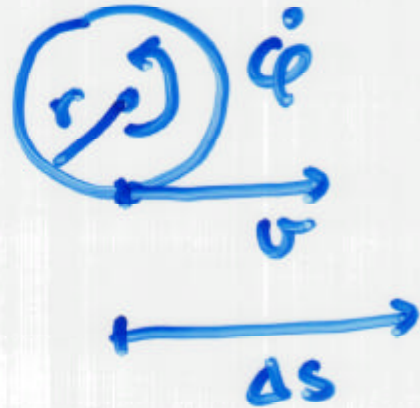
$$v_e = r \dot{\phi}_s$$

$$v_z \Delta T = \Delta s_z$$

$$v_e \Delta T = \Delta s_e$$

$$\Delta s_z = \int_{t_{k-1}}^{t_k} v_z(\tau) d\tau = v_r \Delta T$$

$$\Delta T = t_k - t_{k-1}$$



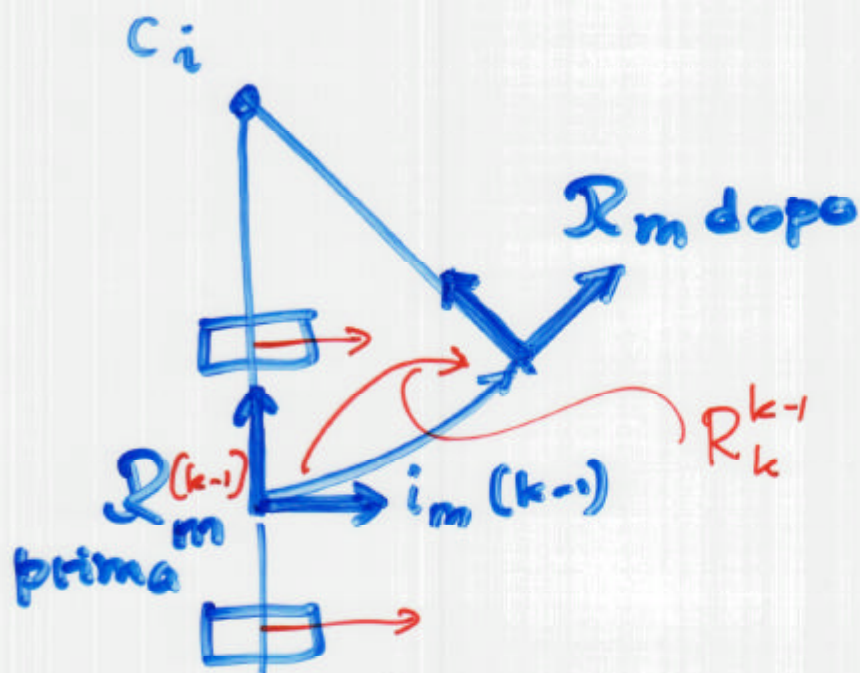
$$p_i \rightarrow \infty$$

per

$$v_z - v_e \rightarrow 0$$

$$v = \frac{v_z + v_e}{2}$$

tra $k-1$ e k ΔT



CINEMATICA DIRETTA

AL TEMPO $k-1$
CONOSCIAMO

$$T_m^0(k-1) \begin{cases} R_m^0(k-1) \rightarrow \theta(k-1) \\ t_m^0(k-1) \end{cases}$$

& COMANDI

$$\underline{v}_z(k)$$

$$\underline{v}_e(k)$$

$$R_m(k-1)$$

$$\underline{v}_z = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} v_z$$

$$\underline{v}_e = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} v_e$$

CALCOLARE

$$T_m^0(k)$$

$$R_m^0(k) \rightarrow \theta(k)$$

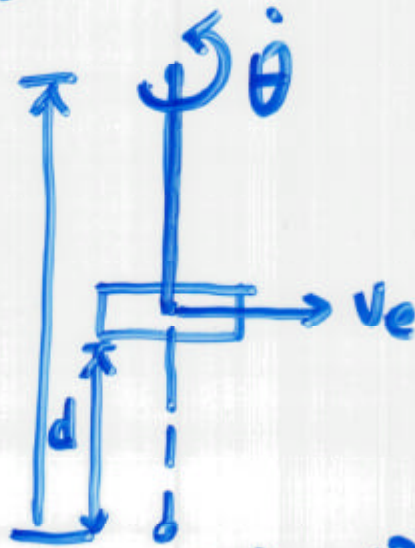
$$t_m^0(k)$$

5

$$v_e(k) = r \dot{\varphi}_e(k)$$

$$= (\rho - d) \dot{\theta}$$

$$= (\rho_i(k) - d) \dot{\theta}(k)$$



$$v_z = r \dot{\varphi}_z(k) = (\rho_i(k) + d) \dot{\theta}(k)$$

$$v_z + v_e = 2\rho_i \dot{\theta}$$

$$\rightarrow \begin{cases} \dot{\theta} = \frac{v_z + v_e}{2\rho} \\ \rho = \frac{v_z + v_e}{2\dot{\theta}} \end{cases}$$

se $\dot{\theta} = 0$ $\rho \rightarrow \infty$

$$v_z - v_e = 2d \dot{\theta} \rightarrow \dot{\theta} = \frac{v_z - v_e}{2d}$$

se $v_z = v_e$ $\dot{\theta} \rightarrow 0$

$$v_o = \frac{v_z + v_e}{2}$$

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METODO A

esatto, ma dipende da p
 e non si può usare se $p \rightarrow 0$

METODO B

approssimato, si può usare sempre

— o —

METODO A vale solo se $v_z \neq v_e$
 o meglio se

$$v_z - v_e \geq \varepsilon \text{ assegnato}$$

per
 $k \geq k-1$

$$R_{m \times m}^0(k-1) \text{ e } \underline{p}(k-1)$$

$$1. \text{ Calcolo di } \dot{\theta}(k) = \frac{v_z(k) - v_e(k)}{2d}$$

$$2. \text{ Calcolo } p_i(k) = \frac{v_z(k) + v_e(k)}{2\dot{\theta}(k)}$$

$$3. \text{ Calcolo } \Delta\theta(k) = \dot{\theta}(k)\Delta T$$

4. Calcolo matrice

$$R_k^{k-1} = \begin{bmatrix} \cos \Delta\theta(k) & -\sin \Delta\theta(k) & 0 \\ \sin \dots & \cos \dots & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

5. Calcolo

$$R_m^0(k)$$

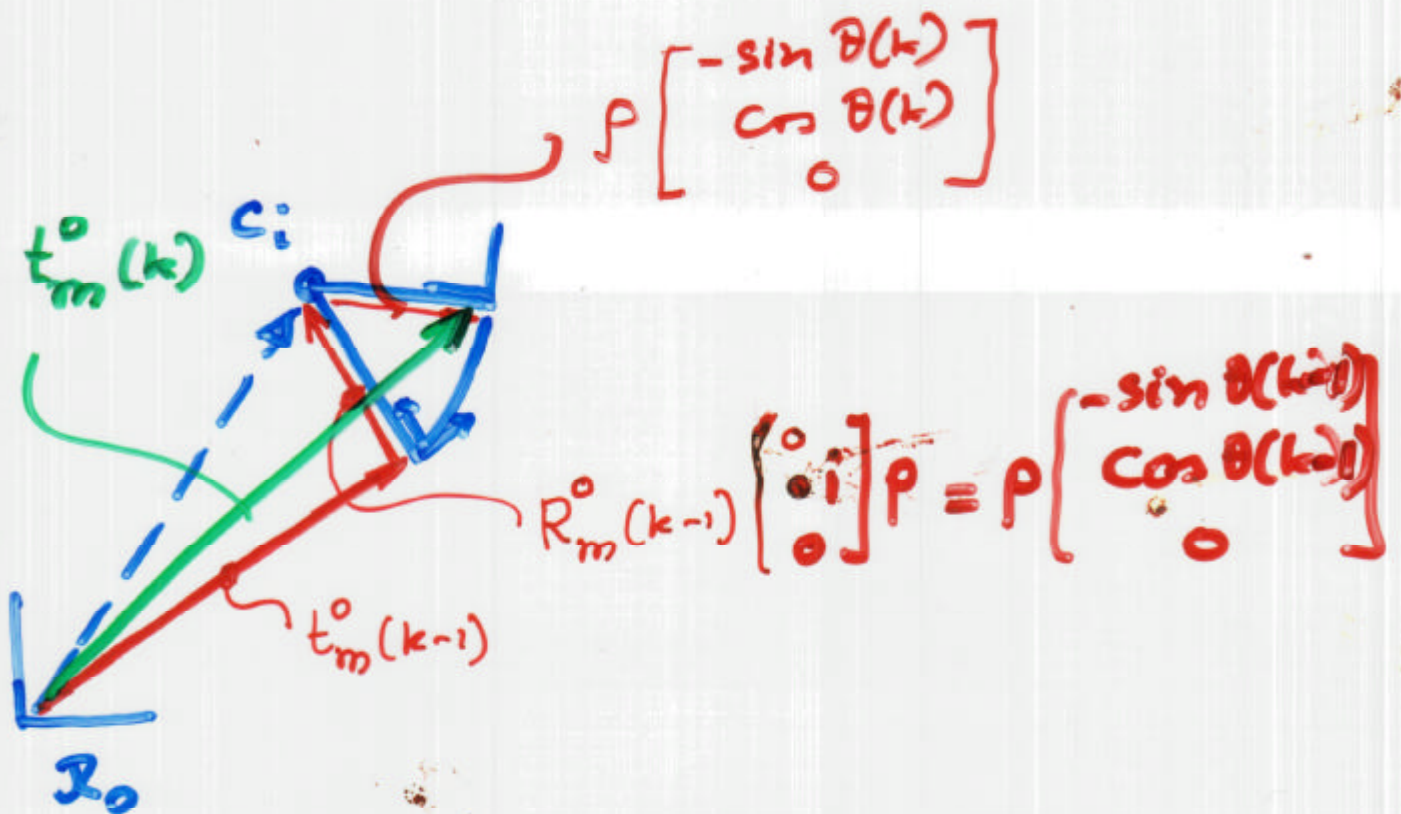
a) calcolo $\theta(k) = \theta(k-1) + \Delta\theta(k)$

$$R_m^0(k) = \begin{bmatrix} \cos \theta(k) & -\sin \theta(k) & 0 \\ \sin \theta(k) & \cos \theta(k) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

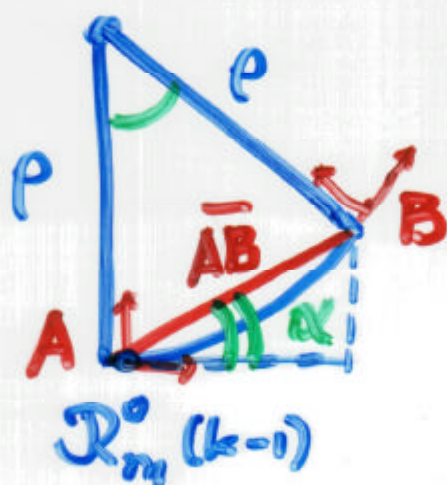
b)

$$R_m^0(k) = R_m^0(k-1) R_k^{k-1}$$

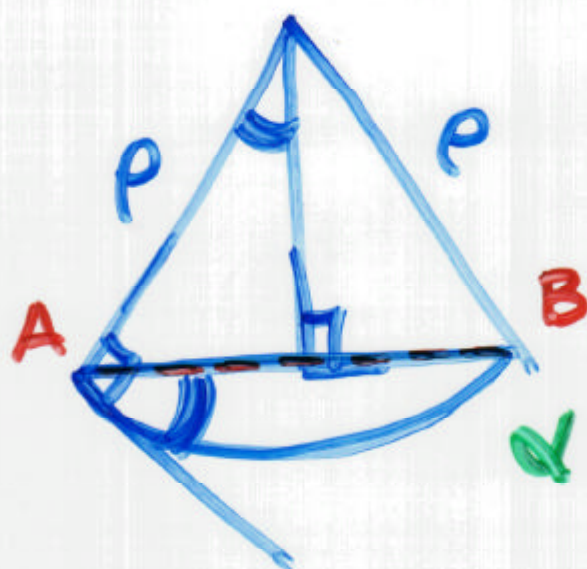
c) $\underline{c}_i(k-1) = \underline{c}_i(k)$



8



$$\vec{A'B'} = \begin{bmatrix} \cos \alpha \\ \sin \alpha \\ 0 \end{bmatrix} \vec{AB}$$



$$\alpha = \frac{\Delta \theta(k)}{2}$$

$$AB \cong \overset{\frown}{AB}$$

$$\text{Arco } AB \doteq \overset{\frown}{AB}$$

$\approx \text{ di } \Delta T \text{ arco } \overset{\frown}{AB}$
 APPROSSIMAZIONE È
 BUONA

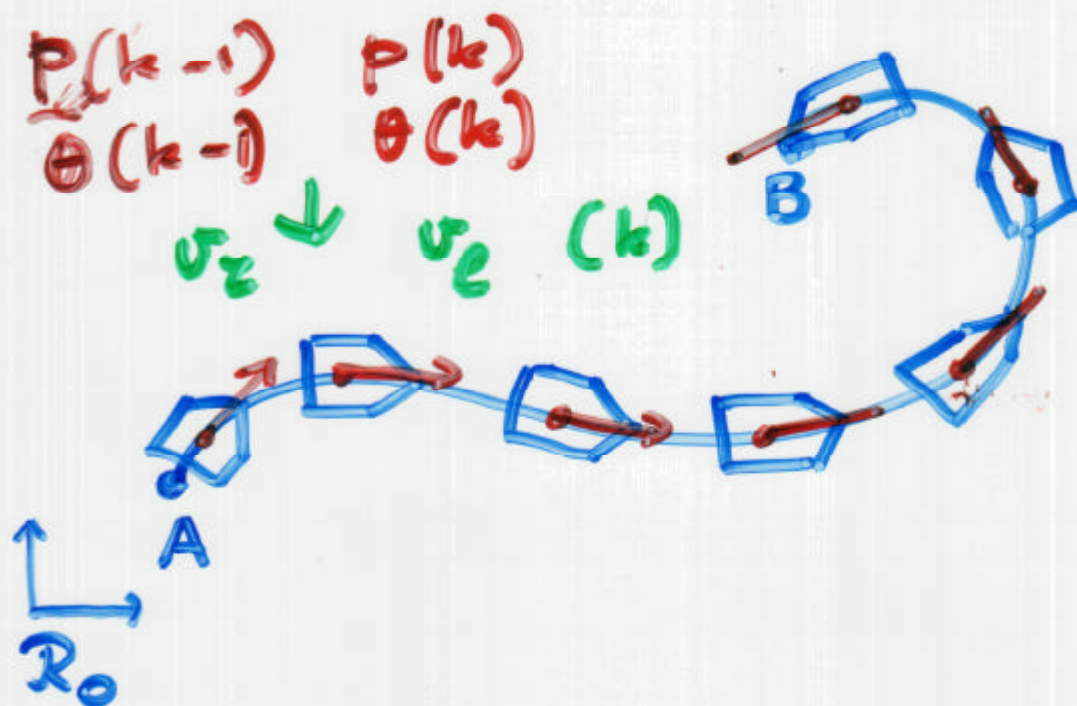


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METODO B

- 1) calcolo di $\dot{\theta}(k)$ con A
- 2) " di $\Delta S(k) = \frac{v_z(k) + v_z(k)}{2} \Delta T$
- 3) $t_k^{k-1} = AB = \begin{bmatrix} \cos \frac{\Delta \theta}{2} \\ \sin \frac{\Delta \theta}{2} \\ 0 \end{bmatrix} \Delta S(k)$
- 4) $t_m^0(k) = t_m^0(k-1) + R_m^0(k-1) t_k^{k-1}$

CINEMATICA INVERSA



$$\Delta\theta(k) = \theta(k) - \theta(k-1) \rightarrow \dot{\theta} = \frac{\Delta\theta(k)}{\Delta T}$$

$$\underline{v}(k) = \frac{\underline{p}(k) - \underline{p}(k-1)}{\Delta T}$$

$$\dot{\theta} = \frac{v_z - v_e}{2d} \rightarrow v_z(k) - v_e(k) = \dot{\theta} 2d$$

$$= \frac{2d}{\Delta T} \Delta\theta(k)$$

$$v_o = \frac{v_z + v_e}{2}$$

$$v_z + v_e = 2 \|\underline{v}(k)\|$$

chiamiamo $V = \|\underline{p}(k) - \underline{p}(k-1)\|$

$$v_z + v_e = \frac{2}{\Delta T} V$$

$$v_z = \frac{V + d \Delta\theta}{\Delta T}$$

$$v_e = \frac{V - d \Delta\theta}{\Delta T}$$

