$$M(\underline{q})\underline{\ddot{q}} + h(\underline{q},\underline{\dot{q}}) = \underline{u}_c$$

$$\underline{u}_c \doteq \hat{M}(\underline{\ddot{q}}_r - \underline{v}_c) + \hat{h}$$

$$\ddot{q} = M(q)^{-1}\hat{M}(\ddot{q}_r - v_c) - M(q)^{-1}\Delta h$$

$$\Delta h = h - \hat{h}$$

$$E(q) \doteq M(q)^{-1}\hat{M} - I$$

$$\eta(q,\dot{q},\ddot{q}_r,v_c) \doteq E(q)(q_r - v_c) - M^{-1}(q)\Delta h$$

$$\text{se } \left.\begin{array}{l} E \to 0 \\ \Delta h \to 0 \end{array}\right\} \to \eta = 0$$

$$\ddot{q} = (\ddot{q}_r - v_c) + \eta(\cdots)$$

$\downarrow$ VARIABILI DI STATO

$$\underline{x} = \begin{bmatrix} \underline{x}_1 \\ \underline{x}_2 \end{bmatrix} = \begin{bmatrix} \underline{q}(t) \\ \dot{\underline{q}}(t) \end{bmatrix} \begin{matrix} n \\ n \end{matrix}$$

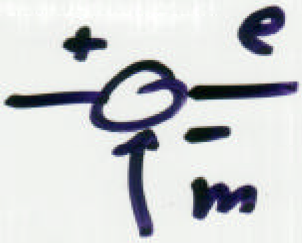
dinamica stato — ingressi — disturbi

$$\dot{x} = Ax + B(\ddot{q}_r - v_c) + B\eta(\cdots)$$

con

$$A = \begin{bmatrix} 0 & I_{n \times n} \\ 0 & 0 \end{bmatrix} ; \quad B = \begin{bmatrix} 0 \\ I_{n \times n} \end{bmatrix}$$

grandezze errore



$$e(t) = q_r(t) - q(t)$$
$$\dot{e}(t) = \dot{q}_r(t) - \dot{q}(t)$$
$$\ddot{e}(t) = \ddot{q}_r(t) - \ddot{q}(t)$$
$$\Downarrow$$

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} = \underline{\varepsilon} = \begin{bmatrix} e \\ \dot{e} \end{bmatrix} \longrightarrow \boxed{\dot{\varepsilon} = A\varepsilon + Bv_c - B\eta(\cdots)}$$

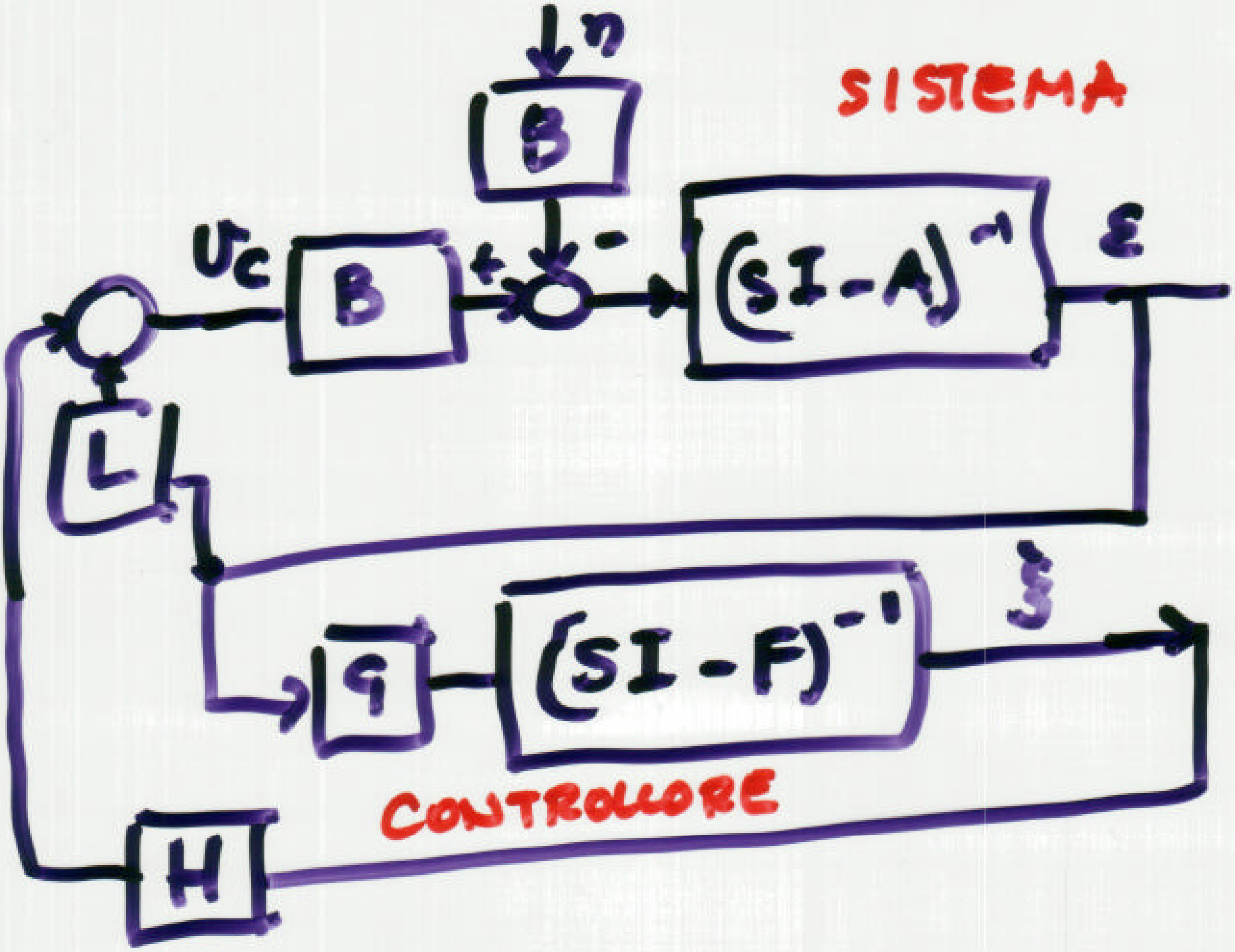
STATO "ERRORE"

(1) $\dot{\varepsilon} = A\varepsilon + B v_c - B\eta$

chi produce v_c ?

è un sistema dinamico "controllore"

$$(2)\quad \begin{cases} \dot{\xi}(t) = F\,\xi(t) + G\,\varepsilon(t) \\ v_c(t) = H\,\xi(t) + L\,\varepsilon(t) \end{cases}$$



$$A = \begin{bmatrix} 0 & I_n \\ 0 & 0 \end{bmatrix}_{12 \times 12}$$

autovalori → poli

6 catene di doppi integratori

INSTABILE

2 ANELLI

(NONLINEARE)

1° è quello che linearizza

$$u_c \doteq \hat{M}(\ddot{q}_r - v_c) \cdots$$

INNER LOOP

2° lineare che stabilizza

$$\begin{cases} \dot{\xi} = F\xi + G\varepsilon \\ v_c = \cdots \end{cases}$$

OUTER LOOP

LINEARIZZ. ESATTA

- $\hat{M} = \hat{M}(q) = M(q)$ COPIA ESATTA STRUTTURA & PARAMETRI

$$\boxed{\eta = 0}$$

- $\Delta h = 0$

$$\Downarrow$$

$$\ddot{q} = \ddot{q}_r - v_c$$

$$\ddot{q}_r - \ddot{q} = v_c$$

$$\ddot{e} = v_c \rightarrow \dot{\varepsilon} = A\varepsilon + Bv_c$$

$$A = \begin{bmatrix} 0 & I_n \\ 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ I_n \end{bmatrix}$$

RETE PD

$$u_c \doteq -K_D \dot{e} - K_p e$$

$$= \underbrace{\begin{bmatrix} -K_p & -K_D \end{bmatrix}}_{-K} \underline{\varepsilon}$$

$$u_c = -K \varepsilon$$

$$\Downarrow$$

$$F = G = H = 0$$

$$L = -K$$

$$\boxed{\ddot{e}(t) + K_D \dot{e}(t) + K_p e(t) = 0}$$

$$\Downarrow$$

$$(s^2 I + s K_D + K_p) e(s) = 0$$

STATO INIZIALE

$$\ddot{e} + K_D \dot{e} + K_P e = 0$$

$$\Downarrow$$

$$\dot{\varepsilon} = A_c \varepsilon$$

$$A_c = A - BK = \begin{bmatrix} 0 & I_n \\ -K_P & -K_D \end{bmatrix}$$

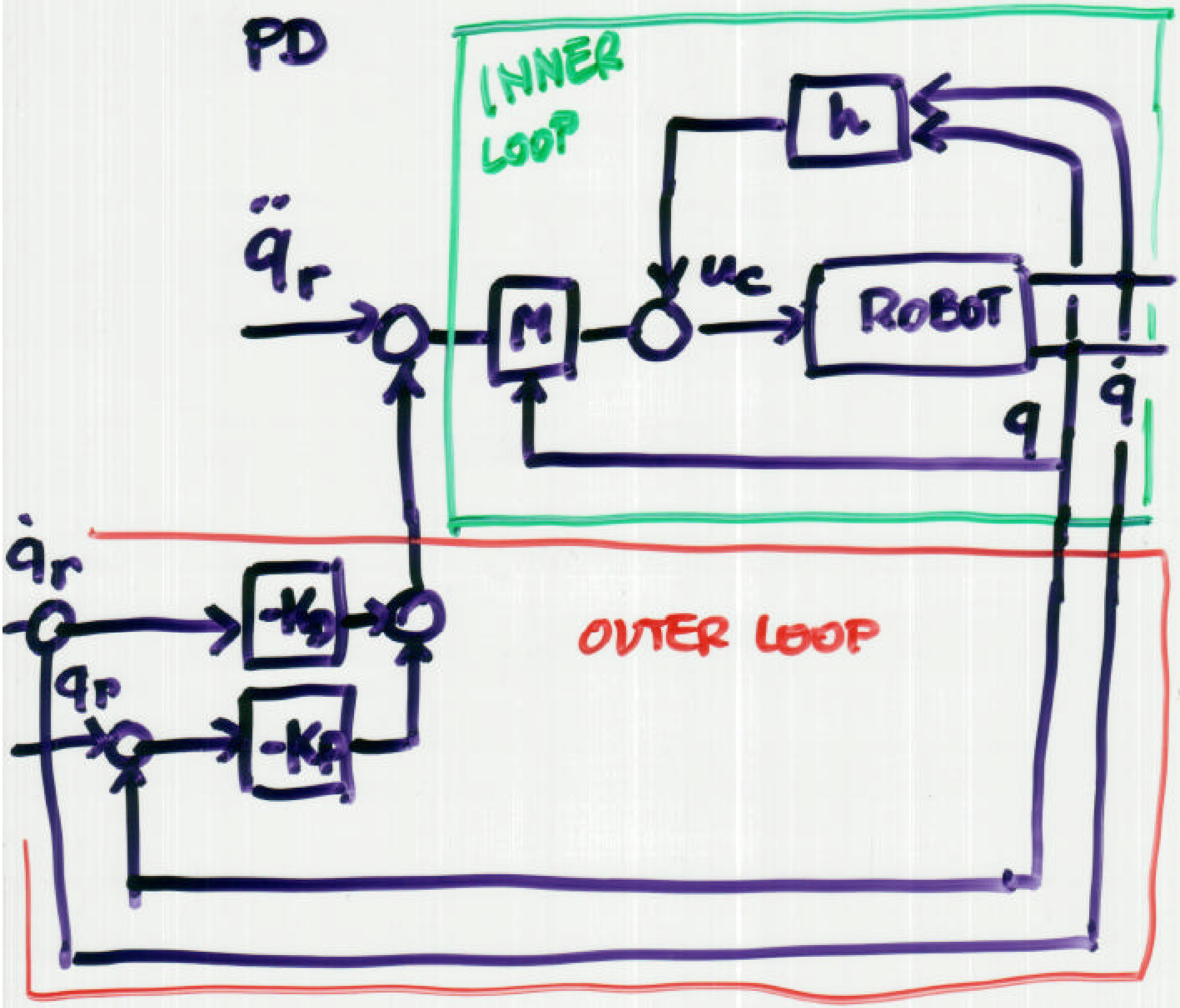
$$\Downarrow$$

$$\dot{\varepsilon} = (sI - A_c)^{-1} \varepsilon_0$$

autovalori
posizionabili
"ovunque"

SCHEMA A BLOCCHI

PD



RETE PID

$$u_c \doteq -K_P e - K_D \dot{e} - K_I \int e\,dt$$

$$u_c = -K_P e - K_D s e - K_I \frac{1}{s} e$$

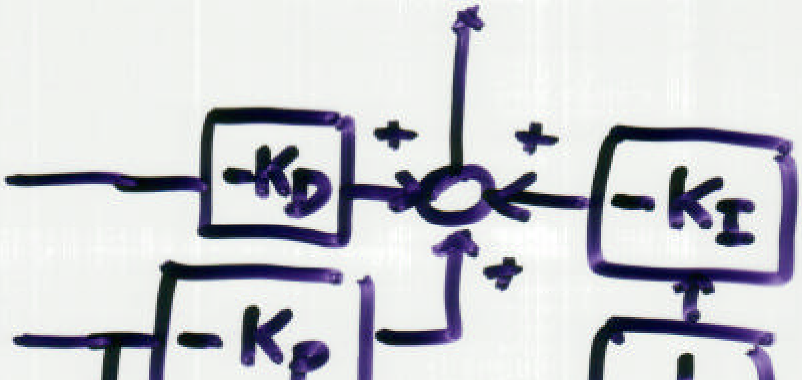
$$\begin{cases} \dot{\xi} = G\varepsilon \\ v_c = H\xi + L\varepsilon \end{cases}$$

Con

$$G = [I_n \quad 0]$$

$$H = -K_I$$

$$L = [-K_P \quad -K_D]$$



$$\underbrace{\left(Is^2 + K_D s + K_P + K_I \frac{1}{s}\right)}_{Is^3 + K_D s^2 + K_P s + K_I} e(s) = 0$$

EQUAZIONE DI STATO SISTEMA + CONTROLLORE

$$\begin{bmatrix} \dot{\xi} \\ \dot{\varepsilon}_1 \\ \dot{\varepsilon}_2 \end{bmatrix} = \begin{bmatrix} 0 & I_n & 0 \\ 0 & 0 & I_n \\ -K_I & -K_P & -K_D \end{bmatrix} \begin{bmatrix} \xi \\ \varepsilon_1 \\ \varepsilon_2 \end{bmatrix}$$

$e = q_r - q \qquad q = q_r - e$

$q_r = e + q$

LINEARIZZ APPROSSIMATA

$$\dot{\varepsilon} = A\varepsilon + B(v_c - \eta)$$

se usiamo PD $\quad v_c = -K\varepsilon$

$$\eta = E(q)(\ddot{q}_r + K\varepsilon) - M^{-1}\Delta h$$

$$\ddot{e} + K_D\dot{e} + K_P e = -w$$

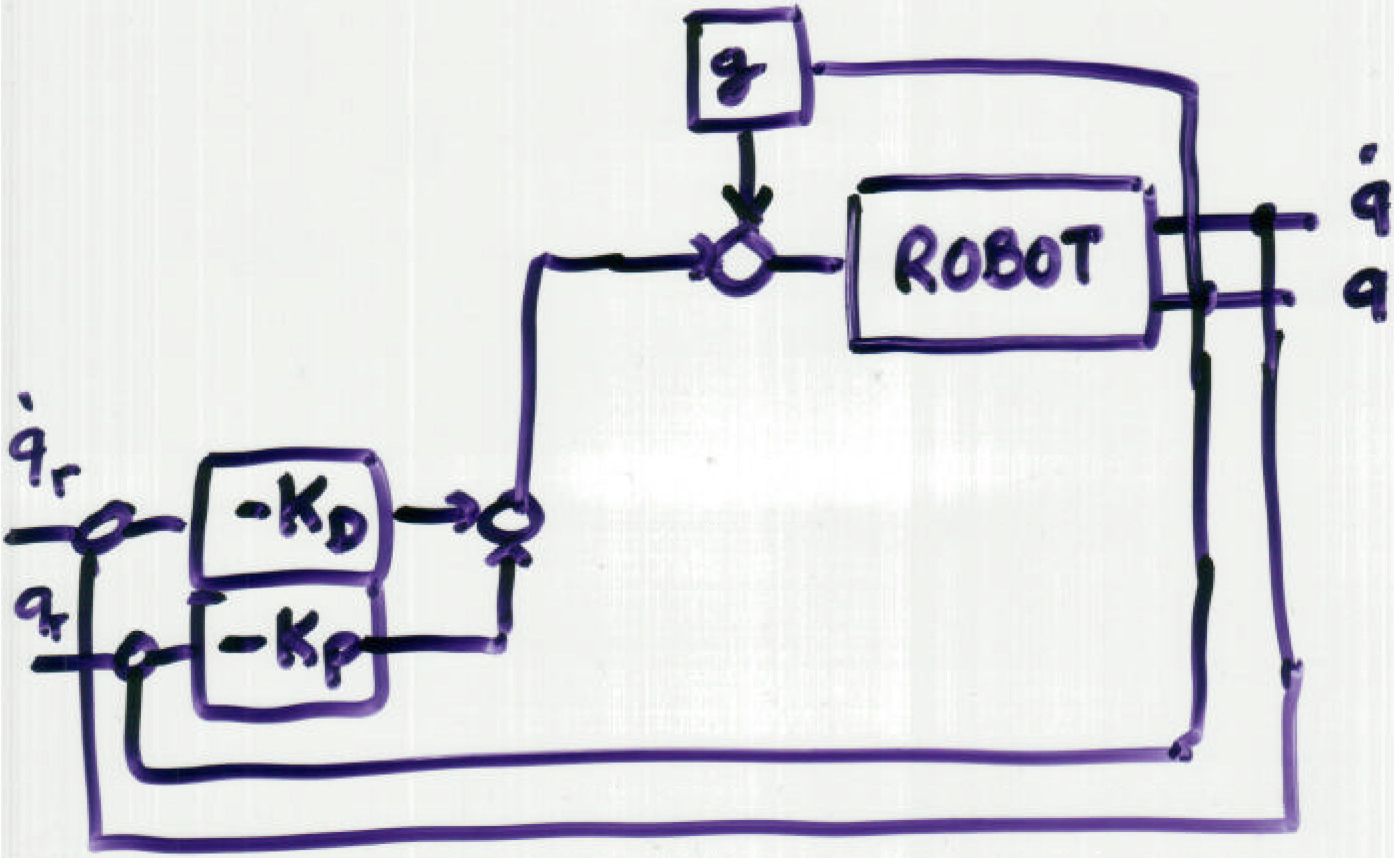
$$\ddot{e} = v_c + Ev_c - E\ddot{q}_r + M^{-1}\Delta h$$

—— o ——

RETE PD con compensazione gravità

$$\hat{M} = I$$
$$\hat{h} = g(q) - \ddot{q}_r \quad + \text{PD}$$

è asintotic. stabile



$$K_D = \begin{bmatrix} \ddots \end{bmatrix}$$

$$K_P = \begin{bmatrix} \ddots \end{bmatrix}$$